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Effects of Minute Contextual Experience on Realistic Assessment of Proportional Reasoning

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Abstract

This mixed methods study describes the effects of a *minute contextual experience* on students' ability to solve a realistic assessment problem involving scale drawings and proportional reasoning. Minute contextual experience (MCE) is defined to be a brief encounter with a context in which aspects of the context are explored openly. The study looked closely at what happened during an instructional unit examining proportional reasoning. Students completed a pretest and posttest involving items characterizing a novel context, and data were analyzed to determine the effects of the MCE. Students were interviewed to gather their perspectives on the problem and their own solutions. Pretest results indicated that instruction in which students demonstrated growth in understanding had little effect on students' ability to solve a novel problem in which they had difficulty associating their everyday mundane knowledge with the realistic context. The students demonstrated a significant increase in ability to solve the novel problem after a MCE. Furthermore, students explicated that the MCE aided their ability to visualize the context, and this helped them apply instructional learning to solve the problem. A discussion of the complexities involving the assumptions of students' familiarity with contexts and their abilities to draw upon their mundane everyday experiences to solve proportional reasoning problems is shared.

Students bring a wide variety of instructional and life experiences, cultural and family influences, and beliefs as they enter class. In addition to what students experience before, they continually have academic and life experiences that shape their being in the world, and these experiences influence the modalities of being (Fleener & Matney, 2007). Within these multitudinous modalities we find particular interest in the way students' experiences of real-world contexts influence their demonstration of mathematical knowledge. In recognizing this variability it is important that mathematics educators continually consider to what degree of certainty students' responses on problems involving real-world contexts reliably indicate the level of their mathematical understanding.

There may be a large number of distinct mathematical connections made by students during a mathematical lesson or unit that transpires over a course of several days or weeks. The way these connections are constructed and the experiential and cognitive sources used to develop them may also differ from student to student. The students' lives compound these distinctive connections as they arrive at the mathematics classroom with a variety of rich experiences (Palm, 2006, 2008), relationships within a variety of cultures, and prior knowledge that impacts their mathematics learning (Boaler, 1993; Schoenfeld, 2011). With these distinct and varied connections it is not surprising that others have expressed careful concern (Boaler, 1993; Carraher & Schliemann, 2002; Palm, 2008) for the influence of context on mathematics learning and students ability to demonstrate extension of mathematics knowledge into novel contexts.

Mathematics instruction that uses problems which characterize a situation may include fanciful, realistic, or authentic scenarios (Lesh & Zawojewski, 2007; Lamon, 2007; Palm, 2006). Realistic problems reference any set of experiences that learners have inside, and more notably, outside of the classroom context. They may be different from situations that commonly occur in the world outside the classroom, but are not foreign to a learner's lived experience (Palm, 2006). For example, students living near a coastline are more likely to perceive a problem involving the beach as a realistic scenario than their peers living in a landlocked area. Students' ability to transfer may be enhanced when teachers employ realistic problems for the purpose of deepening mathematical understanding (Boaler, 1993) and authenticity in the learning of mathematics may emerge for some students through the use of realistic problems (Matney, 2004, 2007). No problem's context is universally familiar and meaningful to all students in the same way; yet some contexts are more familiar and meaningful to one group of students than others (Boaler, 1993; Carraher & Schliemann, 2002).

When engaging students in realistic problems teachers often plan *minute contextual experiences* that give students a brief initial experience with the context prior to working the problem as an attempt to increase its meaning-

fulness and strengthen the possible depth of mathematical connections. A minute contextual experience (MCE) is defined as a brief encounter with a context in which aspects of the context are explored openly. What is the value of such *minute contextual experiences*? In this article we share our exploration into the connections between a teacher's instruction of proportionality, student experiences with context involving a commonly seen set of real-world objects, and a realistic problem used to assess proportional reasoning. There are a few relevant theoretical and research frameworks from which we organized the study to explore the connections among instruction of proportional reasoning, MCE, and assessment. Finally, we offer a brief discussion of these frameworks as they relate to this research.

Theoretical and Research Perspectives

Embodied Knowing

At the core of our study is an embodied cognition perspective (Lakoff & Núñez, 2000; Núñez, Edwards, & Matos, 1999). The connections within and between contexts shape one's perceptions and sense making of the world (Núñez et al., 1999). Knowledge and beliefs influence these perceptions and behaviors (Lakoff & Núñez, 2000; Schoenfeld, 2011). Students' learning outcomes are influenced by their beliefs and academic knowledge as well as their relationships with their environment and other individuals (i.e., context) (Carraher & Schliemann, 2002; Lesh & Zawojewski, 2007; Schoenfeld, 2011). Learning mathematics is an embodied practice that relies on the teachers' and students' prior experiences. Thus, we draw upon an embodied cognition framework that aligns with a perception of knowledge as an internal construct that is heavily influenced by one's contextual experiences.

Proportional Reasoning

We draw on Lamon's (2007) characterization that proportional reasoning supplies "reasons in support of claims made about the structural relationships among four quantities (say a, b, c, d) in a context simultaneously involving covariance of quantities and invariance of ratios or products" (pp. 637-638). A hallmark of one's facility with proportional reasoning is an ability to find a multiplicative relationship between who quantities and extend this relationship beyond those two quantities (Lamon, 2007; Singh, 2000). Unfortunately, many students, especially adolescents, tend to additive reasoning and have amazing difficulty making sense of multiplicative relationships (Singh, 2000). It is critical for mathematics instruction on proportionality to provide students opportunities to reason multiplicatively.

Instruction and Problems

The problems, norms, instructional tools, classroom interactional patterns, and engagement in thinking mathematically contribute to students experiencing positive learning outcomes (NCTM, 2007). It is furthermore important that the problems align with the other facets of instruction if intended outcomes are to be assessed. For example, providing a rich problem that provides the possibility for learning concepts and developing procedures requires students to perceive the affordances of this type of instruction, appropriate social and socio-mathematical norms, and knowledge of how to engage in effective mathematical discourse (Gee, 2008). When the learning environment supports such problems, students are apt to experience improved problem-solving performance and mathematical behaviors compared to experiencing instruction involving exercises, lecture, or mostly teacher-student discourse involving closed-ended questions (Bostic, 2011; Verschaffel, Greer, & DeCorte, 1999). Problems drawing on contexts familiar to students support their mathematics achievement and increase their likelihood for appropriately engaging in a problem (Bostic, 2011; Palm, 2008).

Context

Research on students' learning through experiences with problems drawing on familiar contexts provides evidence that their mathematization of various contexts promote meaningful learning (Bostic, 2011; Carraher, Schliemann, & Brizuela, 1999; Palm, 2008; Verschaffel et al., 1999). Creating such problems and offering them within a mathematics lesson or assessment takes careful planning because it is necessary to determine what is realistic and relevant to students from a community (Bostic, 2011; Lipka, Hogan, Webster, Adams, Clark, & Lacy, 2005; Palm, 2008; Wernet, Lepak, Seashore, Nix, & Reinholz, 2011). Problems drawing on familiar contexts to engage students in mathematizing and making sense of their world are likely to (re)generate cognitive connections among students' knowledge structures. Prior research in this area has shown the benefits for students, yet there are still unresolved questions about students' experience in a learning environment using such problems.

Problems may not replicate real life, but when instruction connects to realistic contexts it highlights opportunities to link academic and everyday mathematics. It is commonly considered that "Problems are realistic to the extent that they typify those encountered in mundane life situations" (Carraher & Schliemann, 2002, p. 136). Familiar contexts within problems impact how the learner approaches and engages with it (Brenner, 2002; Civil, 2002). When the problem's context is meaningful and/or perceived as realistic, then students draw on more cognitive connections to solve the prob-

lem (Palm, 2008). Thus, teachers should provide opportunities for students to contextualize mathematics from a variety of experiences.

Children seem to disconnect their knowledge of real-world context from the mathematics classroom when the two environments differ (Palm, 2008). Palm examined students' reasons for providing unrealistic responses to realistic problems through problem-solving measures and interviews with 161 fifth-grade students. The applied problems were updated from multiple prior studies (see Verschaffel, Greer, & De Corte, 2000) to better simulate genuine context familiar to students. Students' were asked to explain why they provided solutions that may have been unrealistic. Results indicate that modifying a problem's text alone positively influences students' use of real-world knowledge to solve problems. This suggests a much graver concern: learners disconnecting their real-world knowledge when they enter the mathematics classroom. This leads to more unrealistic responses, making further considerations necessary. Palm's study provides evidence about the influence of real-world contexts on students' problem solving but lacks descriptive evidence of the problems' relation to students' lived experiences, and there is no connection to instructional contexts.

Building on Palm's (2008) research, this investigation intends to provide evidence that will give greater understanding about the relationship between instruction, student experiences, and an assessment problem on proportional reasoning. Toward this aim we examine the following three questions:

RQ1: How does instruction using problems drawing on familiar contexts influence students' proportional reasoning performance on a scale drawing problem?

RQ2: Does instruction designed to foster specific contextual connections between experiences inside and outside the classroom impact students' performance on a proportional reasoning problem?

RQ3: What factors do students perceive as influential to their performance gains?

Method

The study was conducted over a 68 day period as delineated by Figure 1. The research began with observations of students and teaching during a 15 day instructional period followed by the administration of the first assessment problem on proportional reasoning. After this, we waited 50 full days before the MCE was observed to allow for memory fade from the assessment. Lastly, the administration of the second assessment problem on proportional reasoning was given followed by interviews of students. These elements of the research flow are more fully developed in what follows.

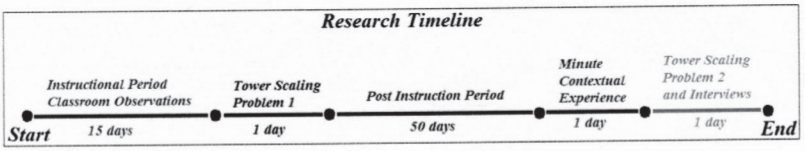

Figure 1. Research timeline showing the flow of activities involved in the study.

Participants and Instructional Context

The participants in the study were from an urban middle school in the south-central United States and consisted of 111 students in the 7th grade and their teacher. The students were 46% female and 54% male. The teacher, Ms. Sights (pseudonym) is a young Caucasian female in her second year of teaching. Ms. Sights and the students in this study were in the second year of a three-year loop. The teacher gave students proportional reasoning problems in which students could use manipulatives, scale drawings, large sheets of paper, and measurement. Several of the problems given during the time of instruction involved scale drawing of visualizing items across scale. The scaling problems that were given to students during the instructional days include, Big Foot and You, How Big is Barbie (Rhodes, 2012), Body Drawing Problem, If We Jumped Like Frogs, and exploring overhead transparency shadow lengths. Students also developed solutions and made sense of problems that did not involve scale drawings. For a more detailed viewing of example problems that were given during instruction see Appendix A.

Ms. Sights challenged the students to make mathematical connections and solve problems involving proportional reasoning through scaling problems such as The Body Drawing problem. In this problem each pair of students outlined one of their bodies on a large sheet of paper and then took measurements of height, neck, arm, head, and foot. The problem directive was to use these and other measures deemed valuable by the students to redraw, as accurately as possible, the entire body on a single sheet of standard construction paper. After working out their solutions students explained and justified their mathematical methods to the class, as this was their typical socio-mathematical norm.

A typical class period was 45 minutes in length and began with a class starter problem involving ratios and proportions or an imagery problem such as Quick Draw (Wheatley, 2007) followed by a time of sharing and discussion of student ideas and solutions. The teacher then launched the day's problem(s) and allowed pairs of students to think and discuss the context of the problem. Students were then expected and allowed to develop methods in which to solve the problem(s), and class ended with a time of sharing and discussion of the mathematical solutions and connections students had made. The teacher believes that when students are allowed to

generate ideas and then discuss possible mathematical connections associated with the problem they find significant meaning in both their engagement with the context and their own solution methods (Boaler, 1993). Every problem involved examining proportional relationships.

The teacher used an approach similar to problem-centered learning (Wheatley, 1991) where students construct mathematical meaning by solving problems, presenting ideas, and learning from one another's solution methods. Observations of the teaching and learning during this unit are filled with students coming up with different approaches to the problem, different mathematical representations, and peer-to-peer discussion of ideas.After the students had completed several problems the teacher began to bring in formal notation for proportions, and the students discussed the meaning of the notation. Students were expected to make sense of notation and use it meaningfully in discussion.

School Context

The school is an inner-urban school with a large at-risk population located in a metropolitan area of more than 1.2 million residents. The school is located in a section of the city with a historically low economic level as indicated by an 86% free and reduced lunch rate. The ethnic diversity at the school from largest population to lowest is 73% Hispanic, 15% Caucasian, 7% Native American, and 5% African American. The administration and teachers work hard to develop a special family-like culture in the school and incorporate the local community in overcoming the life issues at-risk students face that often dampen their chances of being successful academically. Some examples of these life issues as noted by the administration, faculty, parents and students are gangs, abuse, malnutrition, violence, work, pregnancy, drugs, and depression. The administration and the faculty work long hours to provide a context for a community of learners that goes beyond the walls of the school building.

Data and Analysis

Research question one investigated the relationship between instruction using problems that have realistic contexts in the study of proportional reasoning and student success on a scale drawing assessment problem. Data were collected over the 15 days of the instructional unit. Descriptive data were taken from classroom observations with specific attention to presentation discourse, teacher unit plan, teacher reflections, ongoing teacher interviews, and student work. The data were collected and analyzed for themes related to student learning during instruction of proportional reasoning and scale drawing problems (Hatch, 2002). Upon the conclusion of the instructional period the students were given an assessment problem (Tower Scaling Problem 1) involving scale drawing and proportional reasoning. The

context of the assessment problem was purposefully created as different from the contextual explorations during the instruction so as to investigate student ability to use relations from their mathematics learning to solve novel problems. The teacher was not allowed to see the assessment problem prior to administration. The assessment problem was examined by one mathematician and one mathematics educator not affiliated with the study, and they found the problems were appropriate for this grade level and addressed proportional reasoning topics. The students' solutions were evaluated with a scoring rubric (see Appendix B) constructed according to the guidelines given by Moskal and Leydens (2000). Tower Scaling Problem 1 read as follows:

> A vertical tower that is 24 feet tall has a support wire going from the top to a point 10 feet from the base. How long should the wire be? Construct a scale drawing of the situation and use a ruler to assist you in obtaining your answer.

We noted that the problem draws on familiar contexts and within the purview of students' daily lives as two very large towers with many support wires can be clearly seen from any location on the school property. The assessment problem was also developed knowing the sequence of planned instruction on proportional reasoning and that a significant amount of time would be spent on scale drawing type problems.

Research question two looks at the impact of a MCE on students' performance on the assessment problem. More specifically, we wondered if a brief experience with something similar to a tower and support wire context would allow the students to demonstrate a greater knowledge of proportional reasoning and problem solving vis-a-vis scale drawings. Students were engaged in an experience that is in between their mundane everyday experience of the context and the heightened intellectual activity associated with direct problem solving in a particular context. Such an experience was brief in time, allowed for developing a better sense of the context in a non-problem solving way, and had no explicit associations related to mathematical content or processes to be assessed. Teachers often use these short interludes within their lessons to offer additional insight, motivation, and real world connections for the mathematics involved. The MCE was given 50 days after the unit of instruction and administration of Tower Scaling Problem 1 to allow for sufficient time to posttest with Tower Scaling Problem 2 for the purpose of observing possible effects on assessment of students' mathematical knowledge.

Protocol for MCE.

Engaging the students in a MCE relevant to this assessment means exposing them to a context similar to the tower and support wire problem,

discussing the context, its elements, and purpose, yet not conveying associates of proportional reasoning and scale drawing. We worked closely with Ms. Sights to develop a protocol for the specific MCE. She decided to give the students a similar experience to the tower and support wire context using the telephone pole and support wire that was located on school property, about 60 feet from the classroom. The actions of the teacher in launching and facilitating the discussion of the context for each class session followed an agreed upon protocol. Ms. Sights suggested that most teachers would not likely have more than five minutes to engage students in a contextual discussion. Consequently, she desired to keep the MCE less than five minutes. The protocol allowed for the following:

1. The teacher may take the entire class of students outside the classroom for no more than 5 minutes to observe and discuss the context of a power lines, poles, and support wires.

2. The teacher may ask "Why do we have power line and telephone poles? Why is the support wire there? What kinds of things are involved with keeping power and phone lines from harming people? What kinds of mathematics do you think are involved for those building and maintaining these structures?"

3. Researchers would video each of these discussions and check for consistency. One day after the MCE the students were given a re-administration for the assessment problem (Tower Scaling Problem 2) with only the numbers changed. Tower Scaling Problem 2 read as follows:

> A vertical tower that is 15 feet tall has a support wire going from the top of the pole to a point that is 8 feet from the base of the pole. How long should the wire be? Construct a scale drawing of the situation and use a ruler to assist you in obtaining your answer.

There were 51 days between the two administrations of the tower and support wire assessment problems. There was no further instruction on proportional reasoning or scale drawing between the first and second administrations of the assessment problem. The scoring rubic (see Appendix B) was used to assess the students' solutions.

Quantitative methods used to analyze results of these scores include right-tailed *t*-tests for a positive mean increase. An Anderson-Darling normality test (Anderson & Darling, 1952) performed on the paired differences resulted in a *p*-value of 0.005 indicating that these individual differences are approximated fairly well by a normal distribution. With a sample size of 111 we conclude that the distribution of sample means is approximated well by a normal distribution, satisfying the requirements for using a *t*-test. Minitab was used to perform the statistical computations.

Research question three investigated students' perceptions about the problem and what students believe contributed to their improvement. After

completing Tower Scaling Problem 2, students were given a semi-structured interviewed where they were asked to explain how they worked the problem. The answers were recorded and analyzed by looking for themes across all responses. These themes were coded for purposes of matching descriptions to quantitative decline, no change, or growth from the assessment problem. Students who showed the largest growth (a shift of 3 units on the rubric) were subsequently given follow up interviews about their improvement on the assessment. The interviews were transcribed and analyzed for themes related to the students' beliefs about why they were able to solve the problem with more success than before. We read the transcripts, looked for common ideas, created general themes from the ideas, reexamined the data to determine whether there was support for every theme, modified and deleted themes, and retained those themes with sufficient evidence.

Results

Research question one investigated the influence of instruction on student performance on a realistic assessment problem, Tower Scaling Problem 1, which the students had not considered during their instruction. During the instruction there was an observable difference at the class level, of the students' engagement in the problems. Students appeared more intrigued and persevered longer through problems that involved items from their personal lives or used some part of their body. Students appeared to be most engaged by problems that blended what might be considered real, such as the ratio of ones height to foot length, with something fanciful, such as how tall Bigfoot must be. The students' willingness to present their ideas about these problems that connected real worldliness with the legendary or pretend noticeably increased.

Throughout the period of instruction there was ebb and flow between contextual mathematical considerations and inter-mathematical connections. These two delineations appeared in our classroom observations, teacher reflections, and student presentations. Pairs of students presented their ideas after they engaged with the problem and indicated they were ready for discussion. During these presentations students would indicate how they solved the problem and would incorporate the contextual connections necessary for demonstrating the solution. After the presentation the presenters would ask if anyone had any questions or comments about their solution. During this time we noticed that most discussion questions were either contextual mathematical or inter-mathematical. Of these two types, 45% were mathematical questions focusing on the context, and 55% were inter-mathematical questions. Furthermore, none of the contextual mathematical questions and about two-thirds of the intermathematical questions were asked by the teacher. Examples of the two question types can be seen in Appendix C.

Our analysis of the questions asked during presentations revealed that when mathematical questions focusing on the context were asked, they involved issues of going from a large object and shrinking it down in scale to a smaller object. As students presented ideas there was general agreement about solutions that involve scaling from small to large or going from small numbers to large numbers. Students did not ask questions about their peers' thinking in cases where there was a multiplicative growth and in most cases gave feedback to the presenters that there was a large amount of agreement. However, when the problem appeared to need a scaling factor that would decrease the object's size or number's magnitude, the students and the teacher asked the presenters several questions. We found the questioning and answering to be intrinsic to the flow of the instruction, both in the way it shaped the teachers curricular decisions and the way it revealed what ideas the students wanted and needed to spend more time on.

The observations during the instructional period also revealed that many students found difficulty solving problems involving shrinking scale. Problems that included inquiry into things like "How do you fit a picture of your body, in a proportional way, onto a single sheet of paper?" and "If you know the projected shadow of an object, how can you find the objects length?" took students more than twice as long to solve than problems going from small to large.

These observations of students' problem solving and questioning during instruction align well with the teacher's reflections throughout instruction. She noted on several occasions that students were struggling to solve problems that involve a shrinking scale. Ms. Sights reflects,

> I think they have more problems on the body measurements than anything else. They also had problems when going from the shadow to the object. They could go from the object to the shadow very easily, but when it was going backwards they had difficulty.

She further indicated that students found difficulty on any problem if there was a necessary division that did not work out nicely. Regardless of whether the students encountered the idea in a contextual scenario, scaling scenario, or in thinking about decreasing number magnitude this struggle of students became a primary focus on the instruction time. Students began to demonstrate improved proficiency on problems involving shrinking scale by the end of the instructional period, especially when those problems involved numbers that were not terribly cumbersome and could be computed mentally or with relative ease.

Before assessing the students with Tower Scaling Problem 1, we noted the many opportunities during instruction the student had to solve problems that included scale drawings. The item's context involved a tower and its support wire. From the front of the students' school there are several examples of radio and other towers each with their own supporting wires as well

as a multitude of telephone poles with support wires. While they solved the assessment problem, students decided how they wanted to represent the larger scenario in a smaller way. These choices in part determine the level of difficulty of solving the problem. The problem is made considerably easier if the student's choice of unit could be drawn in one to one correspondence with the larger scenario. On the other hand, the choice of representing the unit in a different way could make the problem more difficult.

The analysis of student solutions on Tower Scaling Problem 1 showed that most students found it difficult to interpret the context and design a valid scale drawing from which they could use proportional reasoning to find a solution. As is shown in Figure 2, only four students provided a correct answer but each of them received a 3 on the rubic (out of 4 possible) since their work did not adequately demonstrate a full understanding or proportional reasoning. Only three students did not attempt the problem.

In a survey question asked by the teacher all of the students indicated that they had not only seen the towers but they knew the support wires provided stability for the towers due to the high winds that often occur in this region. Overwhelmingly however, the students were unable to draw upon this knowledge of towers and support wires to appropriately visualize, draw, and solve the problem using their knowledge and instructional experiences involving proportional reasoning.

Research questions two and three were designed to investigate the effect of the MCE on the students' performance on and perceptions about the assessment problems. Histograms of the scores on the proportional reasoning rubric for the 111 students in the study are included in Figure 2. Most stu-

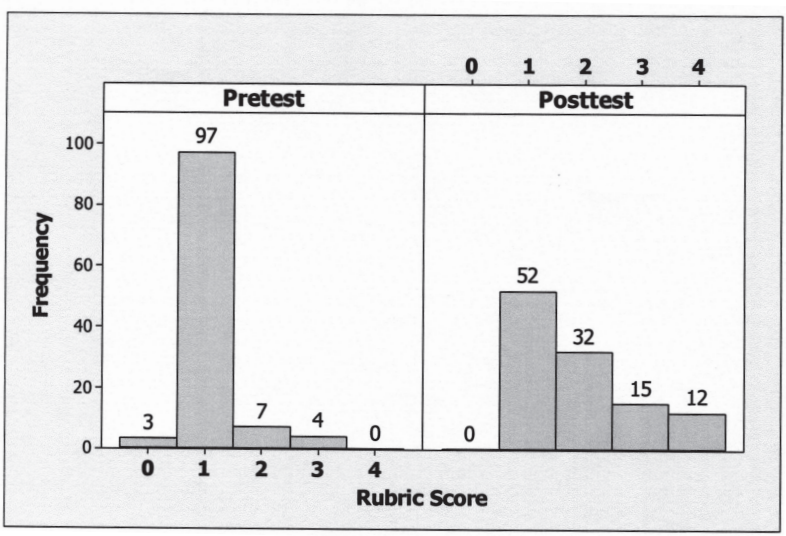

Figure 2. Histogram of Pretest and Posttest Scores.

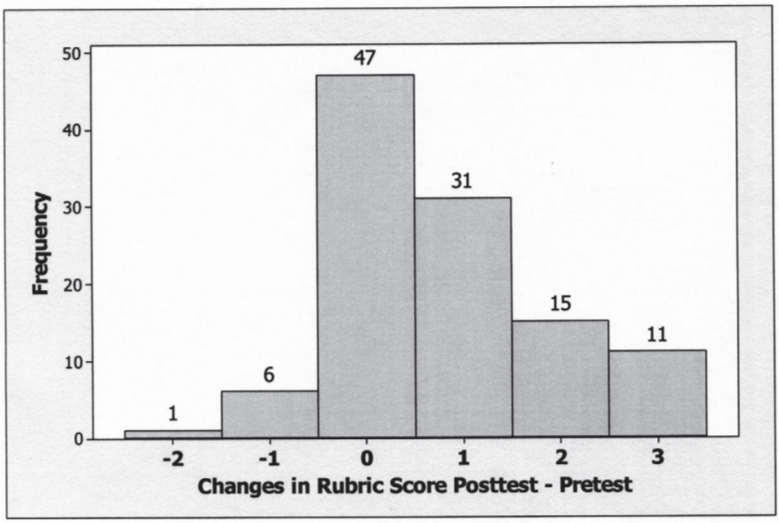

Figure 3. Histogram of Changes in Rubric S

dents (90%) scored either a 0 or 1 on the rubric on Tower Scaling Problem 1 and few scored a 2 thus demonstrating a very low ability to successfully complete the problem. Later on the Tower Scaling Problem 2, 59 students (53%) scored 2 or higher, indicating a better ability to solve the problem. To further analyze the data, pretest scores were subtracted from posttest scores to find the individual changes in score. Notice that positive changes indicate increased ability to solve the problem. Results of these changes are found in Figure 3. Notice that 57 students (51% demonstrated some increase and only 7 students (6%) decreased their scores.

Summary statistics for the changes in score from Tower Scaling Problems 1 to Tower Scaling Problem 2 are given in Table 1. The mean increase in the score is 0.77 and the median increase in score is 1.00. A one-tail paired *t*-test was performed to test for a significant increase versus a null hypothesis of a mean increase of zero. The result from the *t*-test was $p<.001$. There is sufficient evidence to show that the MCE positively influenced students' performance on the proportional reasoning problem.

Table 1.
Scores on the Proportional Reasoning Rubric

Statistic	Pretest	Posttest	Change
Mean	1.11 [1.02, 1.20]	1.88 [1.69, 2.07]	0.77 [0.57, 0.98]
Standard Deviation	0.47	1.02	0.55
Median	1.00 [1.00, 1.00]	2.00 [1.00, 2.00]	1.00 [0.00, 1.00]

After completion of the Tower Scaling problem 2 assessment the students were given a semi-structured interview that began with the following prompts:

- Tell me how you worked this problem.
- Why did you go about it this way?

Analysis of the responses revealed that they fell into one of the following categories. Each response was assigned exactly one of the codes below:

- R: Refused: "I didn't want/feel like doing it."
- DK: Don't Know: "I don't what to draw."
- HP: Hard Problem: "This is a hard problem (I don't know how to do this)."
- F: Forgot: "I know how to do this, but I forgot."
- PO: Picture Only: "I started with a picture, but I am not sure how it helps."
- PR: Proportional Reasoning: "I drew a picture of the context and then related the measures."

Histograms of the changes in scores disaggregated by the responses above are included in Figure 4, and Table 2 includes summary statistics for the changes in score disaggregated by interview category. Figure 5 separates the students into those who had a score increase, those who had no change, and those that had a loss in score. Figure 5 also gives the frequencies of each interview category for each group.

More students explained and demonstrated an understanding of how to connect the Tower Scale Problem 2 scenario to proportional reasoning

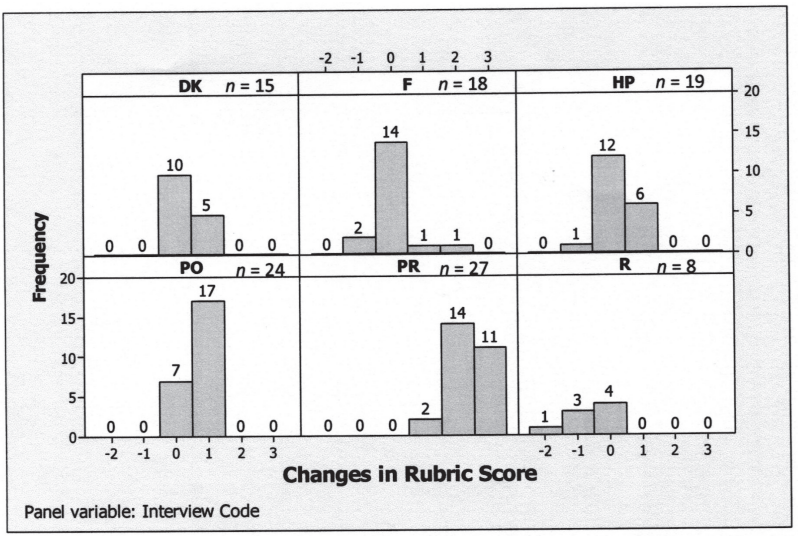

Figure 4. Histograms of Changes in Scores by Interview Code.

Table 2.
Changes in Score Disaggregated by Interview Category

Statistic	Interview Category					
	R	DK	F	HP	PO	PR
Count	8	15	18	19	24	27
Mean	-0.63	0.33	0.06	0.26	0.71	2.33
Standard Deviation	0.74	0.49	0.64	0.56	0.46	0.62
Median	-0.50	0.00	0.00	0.00	1.00	2.00

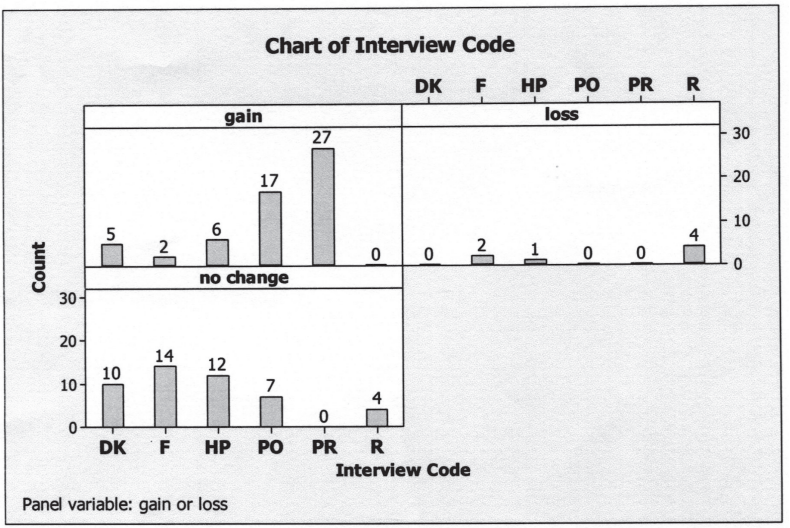

Figure 5. Histograms of Interview Category by Gain or Loss in Score.

(PR) than any other category while the number of students demonstrating proper visualization of the problem (PO) was a close second. The students provided evidence that the MCE helped to shape their visualization of the problem in a way that more of them could be successful. There are a number of students who could not solve the problem (HP and F) and either indicated that the problem was difficult or that they simply forgot how since their last instruction about proportion and scaling was 51 days before. Fifteen of the students indicated that they still were having difficulty visualizing the scenario (DK), even after having the MCE with a similar situation. There is also evidence that some students (n=8) disengaged from the problem (R) and did not persevere in solving it.

There were 11 students who increased their scores from the pretest to the posttest by three units on the rubic. These students were given subsequent

interviews about their perspectives on the assessment problem with their solutions. The questions and flow of the interviews varied from student to student, but the following two questions were asked consistently; "Do you feel confident in your solution to this problem? Why or why not?" and "Why do you think you were able to complete this problem today?"

The interviews with students revealed that their ability to visualize and draw the context made the most difference in their successful completion of the problem and that for these 11 students success came when they could connect their learning and meaning making during instruction with a MCE. An example of this is given in the transcribed excerpt from Margie's interview.

R: Did you feel confident in your solution to this problem? Why or why not?

M: Ya, so the first time we did this a long time ago I thought I got it right, but now I see I got the picture wrong. I know this time it is right.

R: How do you know its correct now?

M: Because it's like the shadow scaling on the overhead. One foot on the shadow may be like only a centimeter on the object. I just drew it [tower and wire] smaller.

R: Why do you think you got it wrong the first time?

M: Mmmmm, I don't know really. I know my proportions because I got a 100 on Mrs. Sights test. So, it's my drawing that messed me up.

Margie explains that she missed the problem the first time because of the way she drew the picture of the context. She also displays confidence that her inability to solve it the first time was not due to a lack of knowledge about proportions since she was successful on Mrs. Sights test. That test was a mixed set of proportions and scaling proportion problems drawing on familiar contexts that had a high degree of alignment and content validity with the mathematics instruction. Margie also appeals to her experience with one of the instructional problems as a connection that gives her confidence that her second solution must be correct. The other ten students also appealed to their instructional experiences during the interview as well as eluded to the value of the MCE in helping them successfully complete the problem. Examples of these connections are demonstrated in the following three interview excerpts from Alondo, Omar, and Mazzielle in their answers to the question, "Why do you think you were able to complete this problem today?"

Alondo's Interview

A: It was easy because it was like the body problem.

B: Tell me more about the body problem. What did you do in that problem?

A: We had to take our body and draw it small, but proportion.

R: How did that help on this problem?

A: I just took the picture from outside and made it small on my paper like the body problem.
R: You had a picture from outside?
A: No, I saw it in my head.

Omar's Interview

O: Because when I read the problem I knew it was a triangle from what was outside.
R: What triangle?
O: This one here [points to paper] like what we saw outside when Ms. Sights asked us to talk about the math. We said there were triangles, cylinders, lines, and rectangles. I saw this problem had that triangle.

Mazzielle's Interview

M: I could really see it in my head.
R: Why?
M: Because when we were talking about it outside we saw it. That helped a lot.
R: What did you see outside
M: All the telephone poles and wires. I drew the picture in centimeters then I just had to measure.

The students discussed how the MCE allowed them to cognitively visualize the context as a "picture" they could "see" in their minds. Similar to Margie's discussion of the transparency shadow problems, Alondo connected the body problem to his successful completion of Tower Scaling Problem 2. Both Margie and Alondo speak about two of the instructional problems that Mrs. Sights allowed the class to think about for a significant amount of time, 2.5 days each. These problems were particularly difficult for the students since it was the first time many of them where challenged to think about the mathematics involved in scaling down.

The excerpts from Omar and Mazzielle serve as examples of how the MCE impacted student visualization of the context and helped them to successfully create a depiction that was helpful in solving the assessment problem. Though during the MCE Ms. Sights did not involve the students in any problem solving, nor did the class mention any connections to the mathematics of proportions the interviewed students all mentioned how experiencing the context, even in a somewhat superficial way, helped them to visualize the context during their solving of the problem. From the interviews students presented evidence that they uniquely connected various elements of their learning during instruction and the MCE.

Summary of the Study Findings

Classroom observations, teacher unit plan and reflections, student work, and our analysis of classroom discourse provided evidence that students'

most significant struggle in their study of proportional reasoning involved problems of scaling from something large to something small. The teacher adjusted to meet the needs of the students and allowed them much time to wrestle with and discuss the ideas. By the end of the instructional period students' discourse and work displayed a much improved understanding of scaling down and proportional reasoning. However, most students were not able to demonstrate this understanding on the novel Tower Scaling Problem 1. Though the students had realistic and every day examples of the context, they still found it difficult to visualize and draw the context in a way that was helpful to solve the problem.

Students' ability to demonstrate the transfer of their proportional reasoning knowledge was diminished by their difficulty envisioning and drawing the context. After the MCE, in which students observed and briefly discussed a context similar to the assessment scenario, they were able to perform significantly better on Tower Scaling Problem 2. The students who showed the most growth perceived the MCE as a beneficial component to successfully completing the assessment problem as it helped them to visualize the problem in more beneficial ways.

Discussion and Complexities

The students in this study had real-world examples of the context in plain view each day of their life at school and had given instruction involving scaling contexts. Moreover, the instruction evolved to focus on the mathematical struggles associated with going from large scale objects to smaller. Students developed and presented mathematical strategies to solve these kinds of problems successfully. Yet the students still found it difficult to visualize the context in a way that would allow them to demonstrate their knowledge of proportional reasoning via scale drawings.

Problem solving is a focused cognitive activity (Schoenfeld, 2011). Though students encounter and experience many things in their lives, such as towers and support wires, the amount of focused thought about these things may be minimal and as such remain cognitively mundane. The difficulty students have in transferring their everyday experience to the solving of judiciously selected realistic problems appears to be related to the amount of thoughtful inquiry they have given to that particular context. Although the context may clearly be familiar to the students, so long as it remains mundane and is not revealed as a cognitive construct for inquiry, the difficulty of association and transference may remain for most students. As demonstrated in this study, this difficulty even occurs for the students who have the mathematical understandings necessary to solve the problem.

The findings of this study support other research findings that demon-

strate how familiarity with a context impacts the way a learner approaches and engages with it (Brenner, 2002; Civil, 2002). The study reveals a deeper complexity than typically found in the common parlance that realistic problems are simply those which best exhibit qualities of mundane life situations (Carraher & Schiemann, 2002). Based on the observed growth of students' understanding during instruction we initially assumed that the students would not likely find the assessment problem to be overly difficult. That assumption was shown to be an error as students had great difficulty connecting their everyday experience of the realistic context to solution methods of the assessment problem. This problematizes the notion that using an assessment context which is clearly within the purview of all students' everyday lives will likely produce a clear picture of students' mathematical knowledge.

To achieve a significant level of meaningfulness in problem solving there appears to be a need for more than a mundane familiarity. In this study students reported a familiarity with the context and likely saw towers and poles and support wires each day of their lives. These experiences may allow the students to be "familiar" with the context in a mundane everyday way, but familiarity may still lack the heightened cognitive experience necessary for connecting the context during problem solving.

These findings are especially relevant for those states that have adopted the Common Core State Standards (CCSSI, 2010) and are working through their respective assessment consortia to assess students' mathematical understandings of proportional reasoning and scale drawings. In Grade 7 of the Common Core State Standards for Mathematics (CCSSM) there is a focus on ratio and proportional relationships. Within this focus there is a direct connection to the role of scale drawings. Scale drawings are mentioned in two of the four critical areas.

> Students solve problems about *scale drawings* by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects…. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using *scale drawings* and informal geometric constructions, and they gain familiarity within the relationships between angles formed by intersecting lines (p. 46).

The importance of the role of scale drawings is further detailed by the first geometry standard in Grade 7 which says, "Solve problems involving *scale drawings* of geometric figures, including computing actual lengths and areas from a *scale drawing* and reproducing a *scale drawing* at a different scale" (p. 49). The CCSM make clear that students should be able to use their mathematical understandings to solve problems that are both real-world and

mathematical. This is mentioned three times in the Grade 7 overview. There is a strong intention to assess students using real-world contexts and assessment designers should be ever cautious in designing these problems as these students, from different states, will likely have more diversity of experience than those students found in this study. Further more, what may be reasonable to consider as realistic for all students based on the contexts seeming familiarity might be nothing more than a mundane experience for some students, which may result in a difficulty of association and transference, not based on their mathematical knowledge, but due to the their experience with the context.

Within the mathematics education community there has been discussion about what kind of contexts found in application problems have mathematical value in out-of-school real-world contexts (Gerofsky, 2006; Palm, 2006, 2008; Stocker, 2006). Do problems that involve the fanciful still hold meaning and relevance for mathematics learning? The instructional problems given by the teacher in this study often blurred the line between fanciful and realistic contexts. From our observations we found that students were more engaged in making sense of proportions with such problems. For example, students persisted longer during the problems that involved the students' bodily ratios and something non-real, like the mythical Bigfoot or the non-living Barbie dolls than they did in other problems like the transparency shadow problems. Moreover, the students' presentations provided evidence that they understood and were more excited to share their mathematical solutions to those kinds of problems. From this we find support for the claim made by others that there can be mathematical value in the fanciful so long as students can relate to the scenario (Geoofsky, 2006; Van den Heuvel-Panhuizen, 2005). When these somewhat fanciful mathematics problems are given *and* students are allowed to approach the problem and develop their own strategies, they are enacting the formulation of mathematical notions in a very real way, within a very real mathematics learning community. This provides a space for developing habits of mind and knowledge conducive to productive mathematical thinking outside of the school.

As per the findings of this study however, the mathematical value does not necessarily extend to realistic assessment problems for which students' experiences of the context remain mundane or appear as disconnected from their community of inequity. This supports Boaler's (1993) assertion that "It is probably safe to assume that transfer is not enhanced by contexts unfamiliar to students, nor by contexts which are perceived by students as another sort of school mathematics" (p. 15). The findings here suggest that we can go a bit further and say that transfer is not necessarily enhanced even when the context may be reasonably considered as a familiar one. So long as the everyday familiar context remains one that is mundane or on the periphery of cognitive inquiry, students have difficulty demonstrating transfer.

Anecdotal wisdom of the need to cognitively engage students in the context before engaging them in problem solving involving that context has lead teachers to use MCE within instruction to inspire students' thoughtful attention toward the problem and to help raise their likelihood of connection making during the problem. When teachers give a MCE it allows students to organize their mathematical thinking involving the context in more productive ways. The findings of this study support teachers' use of MCE as a method for stimulating students' visualization and engagement which may lead to better assessment of students' mathematical knowledge through the solving of problems involving similar contexts.

Through the students showed a significant increase in their ability to solve the Tower Scaling Problems our analysis of student responses, particularly those who still had trouble visualizing the tower scenario (DK) after the MCE, invited us to further consider the possible impact the difference in the sizes of the telephone pole and tower contexts may have had. The telephone pole context was itself a scaled down version of the towers viewable from the school grounds. Could part of the difficulty for the students who continued to struggle with the Tower Scaling problems be found in the difference in sizes between the MCE of the telephone pole and the assessment problem of the tower? Further research needs to be done inquiring into the possible difficulties magnitudes of scale play in students' successful completion of contextual scaling problems.

The results of this study raise other questions about the role of experience and its connection to student understanding and assessment of student mathematical ideas. Small differences in this research could be applied to the student experience or the assessment itself in order to more deeply investigate these findings. For example, does a concrete contextual experience, in which students interact with and discuss the feature of the situation, differ from a semi-concrete experience in which students are given video or photographs of the situation? From the findings of this study we see a need to further inquire about which contexts might require something like a concrete or semi-concrete MCE to allow students to better demonstrate their mathematical knowledge. Lastly, within these explorations of contextual considerations we must look more closely at for whom these kinds of experiences are beneficial and why.

What about the assessment experience itself? What if students had been given the same text-based problem while they were outside experiencing an example of the tower and support wire? Would the outcome on the assessment have been different? For whom would it have been different, and why? Could the same results be achieved from students being provided with a semi-concrete experience via a real world picture of a tower and support wire along with the text, or would the provision of a picture confuse the students even more since they might try to use the provided picture instead

of creating their own scale drawing that makes sense to the problem? We recommend more research be done concerning these issues prior to the use of contextual scaling problems in high stakes testing.

Limitations

The boundaries of the study, one teacher and her students in one school of one grade level, limit the findings. Important questions about the general effect on students' assessment outcomes depending on the teacher and instructional problems cannot be answered by the study. In conducting the two assessment problems, we waited 51 days between administrations to allow for memory fade of the problem. The lengthy duration was necessary but also involves an atypical amount of time between a teacher's instruction and summative unit assessment. On the other hand, in consideration of the state or national level assessments given to students in April and May of each year, this duration may be quite reasonable.

The study involved a close look at how students performed on one scaling problem connected to a particular MCE. The nature of the inquiry with student experiences makes this kind of research time consuming and difficult to do on a large scale with many different contextual scaling problems. Furthermore, scaling problems are not the only problems that can be used to inquire about students' proportional reasoning. Although the teacher's summative exams were not a part of this study it should be mentioned that Ms. Sights explained that the students showed a greater degree of success on her assessment which included both contextual scaling problems and more traditional problems involving proportional reasoning.

The research presented here studied one teacher's instruction of proportional reasoning, the effects of a MCE, and a scaling assessment problem of proportional reasoning. We would like to see similar research projects that focus on the relationship between instruction, MCE, and assessment of mathematical ideas. It will take many such undertakings to gain significant insight into what's possible in both virtual and live educative settings and to better design assessments that allows the student to demonstrate knowledge and prowess in using mathematical relations to solve novel contextual problems.

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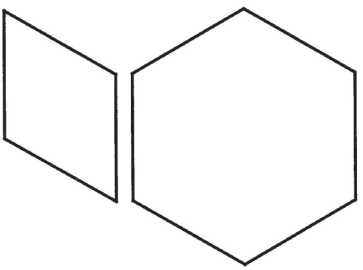
Appendix A

Examples of proportional reasoning scaling problems given during instruction:

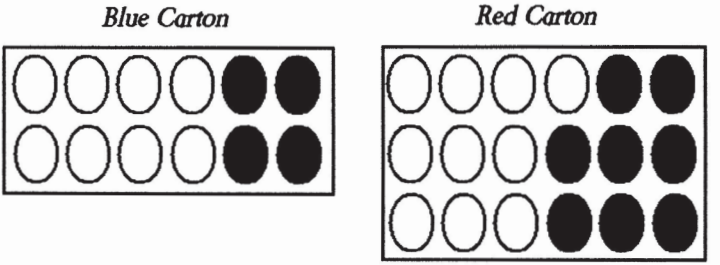
1. Body Drawing Problem- Each pair of students will trace one person's body onto a big piece of paper. Using the traced body you will take measurements of the height, head, neck, arm, leg, and foot. Your task is to precisely redraw the body on a single sheet of construction paper, proportionally. You may add additional measurements as you feel they are necessary.
2. Bigfoot (Legend of Sasquatch) Activity- (Have a scale drawing/cast of Bigfoot's foot for students to measure) If Bigfoot's height and foot length are proportional to your height and foot length, how tall would Bigfoot be? Demonstrate the difference in height with a scale drawing.
3. Overhead Shadow Lengths- We will start today by noting the measures of the pencil on the overhead projector and its shadow on the board. Each pair of students will pick a different object to put on the overhead projector. The criteria are that it must be different from a pencil, different from other groups, and be able to fit on the overhead projector. Your task will be to see if you can come up with a way to accurately predict either the length of the shadow when you know the objects length or the length of the object if you know the shadows length.
4. The following are scale diagrams for two walking tracks. (You may assume that all of the sides are the same length.) It takes John 0.22 hours to walk around track A. How long would it take him to walk around track B?



Track A Track B

Examples of proportional reasoning non-scaling problems given during instruction:

1. Sally bought 3 pieces of gum for 12 cents and Anna bought 5 pieces of gum for 20 cents. Who bought the cheaper gum or were they equal?
2. There are 7 girls with 3 pizzas and 3 boys with 1 pizza. Who gets more pizza, the girls or the boys?
3. When making coffee, David needs exactly 8 cups of water to fill 14 small glasses of coffee. How many small cups of coffee can he make with 12 cups of water?
4. There are two egg cartons. The shaded circles represent brown eggs and the un-shaded circles represent white eggs. The blue carton contains 8 white eggs and 4



brown eggs. The red carton contains 10 white eggs and 8 brown eggs. Which carton contains more brown eggs relative to white eggs?

5. If the ratio of apples to oranges is 2:3, and we have 13 apples, how many oranges would we have? What if we had 22.5 oranges, how many apples would we have?

Appendix B

Proportional Reasoning Assessment Rubric

Score	Description
0	The student does not attempt the problem or what is written is nonsense. No observable evidence exists of understanding.
1	The student obtains an incorrect answer and their solution is either not present or demonstrates a major misconception. Little or no evidence is given showing appropriate understanding of the relationship between the existing or constructed model to the contextual situation of the problem.
2	The student demonstrates some evidence of understanding of the problem but shows limited evidence of understanding either an appropriate strategy or the relationship between the existing or constructed model to the contextual situation of the problem.
3	The student demonstrates some evidence of understanding of the problem and a partial application of an appropriate strategy, perhaps with an incorrect conclusion due to a computational error, OR the student obtains the correct answer through an appropriate strategy but evidence of the student's understanding of the relationship between the existing or constructed model to the contextual situation of the problem is incomplete or faulty.
4	The student obtains the correct answer and their solution shows clear evidence of understanding of the problem and applies an appropriate strategy to its solution. Evidence is provided that the student understands a clear relationship of the existing or constructed model to the contextual situation of the problem.

Appendix C

Examples of contextual mathematical questions from instructional problems found in Appendix A

- How did you use the pencil length to estimate the eraser length?
- So if you were shrunk down to Barbie's size how many centimeters long would your head be?
- What did you do to figure out how long your arm was when you shrunk it down to fit on the page?
- How much longer is Track B than Track A?
- So you looked at how many of your feet would go into big foots to find that number?

Examples of inter-mathematical questions from instructional problems found in Appendix A

- How did you know what to divide by?
- Why does division of those two quantities make sense to you?
- What relationship did you see between the numbers?
- What did you do with the numbers to get 0.055?
- Is your thinking multiplicative or are you doing something else?

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