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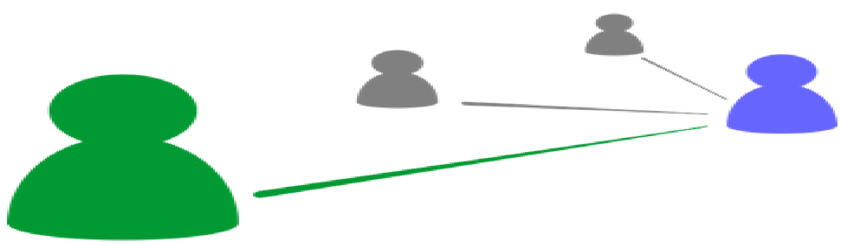

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FOURTH-GRADE STUDENTS' SENSEMAKING OF WORD PROBLEMS

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The purpose of this study was to investigate fourth-grade students' sensemaking of a word problem. Sensemaking occurs when students connect their understanding of a situation with existing knowledge. We investigated students' sensemaking about a word problem by comparing students' strategy use. Inductive analysis was used to find themes about student sensemaking. Students exhibited one of three levels of sensemaking. Some problem-solving strategies, as a result of students' sensemaking, led to a greater frequency of correct results.

Standards represent each states' expectations for what content should be taught. Many states have adopted some form of the Common Core State Standards for Mathematics (CCSSM; CCSSI, 2010). The CCSSM established real-life problem solving as something students should be engaged in throughout their K-12 education (CCSSI, 2010, p. 6, 7, 84). Furthermore, teachers should promote students' mathematical proficiency through providing opportunities for students to "make sense of problems and persevere in solving them" (CCSSI, 2010, p. 6). This study investigates fourth-grade students' sensemaking about a *multi-step situational word problem*, providing the mathematics education community with evidence about students' sensemaking in the Common Core Era.

Theoretical Frameworks: Problem-solving and Sensemaking

This study is framed by notions of problem solving and sensemaking about situational word problems. Broadly speaking, problem solving "is what you do when you don't know what to do" (Sowder, 1985, p. 141). Verschaffel et al. (2000) describes a six-stage model of problem solving that includes (a) reading the problem, (b) creating a representation of the situation, (c) constructing a mathematical representation of the situation, (d) arriving at a result from employing a procedure on the representation, (e) interpreting the result in light of the situational representation [see (b)], and finally, (f) reporting the solution within the problem's context. In consideration of students' sensemaking, we utilize a framework for problems such that the word problems are (a) open, (b) developmentally complex, and (c) realistic tasks for an individual (Verschaffel et al., 1999). Open tasks can be solved using multiple developmentally appropriate strategies. Word problems therefore are mathematical tasks presented as text, which contain real-life situational background information (Verschaffel et al., 2000). We define strategy as the mathematical pathway an individual enacts while problem solving, which includes both representations and mathematical procedures (Goldin, 2002).

Sensemaking about Word Problems

Sensemaking is when students develop an understanding of a situation or context by connecting it with existing knowledge (NCTM, 2009, p. 4). The way students make sense of problems can vary quite a bit due to cognitive, social, and environmental factors (Cifarelli & Cai, 2005). During problem solving, students need to make sense of the word problem by observing connections between the situation being presented and the mathematical representations and operations necessary for a solution (Verschaffel et al., 1999; Verschaffel et al., 2009). The word problem increases in sensemaking difficulty when the situation necessitates more than one operation, and the use of the result from the previous operation must be interpreted and used in the context of a different operation (Quintero, 1983). Sensemaking is essential for successful problem solving (Pape, 2004; Verschaffel et al., 2000). Development of sensemaking habits help students develop autonomy, relying on their own reasoning and resources to be more persistent while problem solving (Meuller et al., 2011; Yackel & Cobb, 1996), and ultimately foster productive dispositions as mathematically proficient problem solvers.

Sensemaking occurs at many steps in the problem-solving process (Verschaffel et al., 2009) and some have focused on students' work between the situation and mathematical stages as a way to explore sensemaking. For instance, Palm's (2008) qualitative study examining fifth-grade students' work indicated that students' engagement with realistic word problems increased the likelihood their problem solving ended with a correct solution to a problem. Similarly, Yee and Bostic (2014) also conducted a qualitative study examining secondary students' word problem solving and drew a conclusion that more successful problem solvers were flexible with their mathematical representations often using non-symbolic representations, compared to others who employed symbolic tools. Taken collectively, the literature provides ideas about students' problem solving but few take a critical look at students' work to explore their mathematical sensemaking of word problems. Hence, this study aims to fill a needed gap within the problem-solving literature.

Method

The Fair Task

This study stems from a broader grant-funded project aiming to develop problem-solving tests that align with the Common Core State Standards for Mathematics in grades 3-6. Each Problem-Solving Measure (PSM) is composed of 15 items addressing grade-level content. Validity evidence has been gathered for each test and led to a robust and valid score interpretation and use arguments (e.g., Bostic et al., 2019). In this study, we investigated students' sensemaking of one purposefully selected word problem from the PSM for grade 4. The Fair Task states, "Josephine sold tickets to the fair. She collected a total of $1,302 from the tickets she sold. $630 came from the adult ticket sales. Each adult ticket costs $18. Each child ticket costs $14. How many child tickets did she sell?" It incorporates multi-step thinking and addresses Operations and Algebraic Thinking (OA) standards. Specifically, students are expected to make sense of a mathematical difference and the number of groups within it. This task was selected because (a) it is of moderate psychometric difficulty for average-performing students, (b) multiple developmentally-appropriate strategies have been used to solve it, and (c) it is connected to standards that are linked with fostering algebraic understanding (Smith, 2014). Through the PSM validation process, the Fair Task was reviewed by mathematicians, mathematics educators, and mathematics teachers. Drawing upon the knowledge of these experts three key observations (KO) to successfully solve the Fair Task were generated. These KOs are tied to sensemaking of various parts within the word problem. (KO1) The difference between $1,302 and $630 is the dollar amount brought in by selling child tickets. This value is $672. (KO2) Each child's ticket is $14. There is some number of groups of 14 that represent the number of child tickets sold. (KO3) The number of groups of 14 within the unit of 672 indicates the number of child tickets sold. We drew upon these KOs to explore two research questions. (RQ1) How do students draw upon sensemaking while solving the Fair Task? (RQ2) What mathematical strategies did students use while problem solving and how were those strategies related to students' successful problem solving on the Fair Task?

Participants and Setting

In total, 280 fourth-grade participants were included in the study. They came from a rural and a suburban school district located in a Midwest state that adopted the CCSSM. The PSM4 was administered near the end of the academic year in paper-and-pencil format. PSM4 administration followed the same practice as usual. Students solved problems individually, in a quiet classroom setting monitored by the researchers and a classroom teacher. They did not use calculators, had up to 120 minutes for test administration, and were encouraged to write, draw, and represent their ideas on the testing paper. Any students named in this proceeding are pseudonyms.

Data Collection and Data Analysis

Participants solved the Fair Task and expressed their strategy use and result from problem solving in writing. The written work on the Fair Task was reviewed by a team of three researchers. This largely qualitative study of students' written mathematical work on the Fair Task used inductive analysis (Hatch, 2002) to generate themes about students' sensemaking. The coding process of analysis had multiple steps. Three researchers read the solutions of all 280 students. The frame for the analysis was evidence of student mathematical sensemaking of the problem related to the three key observations for the Fair Task. Researchers looked for sensemaking as evidenced by student work conveying understanding of the connection between the Fair Task context and students' mathematical strategies for solving the problem at hand. The researchers identified salient domains, clusters of strategy types, and gave them a code. Each researcher took a specific domain and reread all of the students' solutions to decide if the domains were supported by the data. Discrepancies were shared with the research team and discussed for consensus. This completed analysis for RQ1. The authors created written paragraphs and graphic maps to describe each domain. The completed domains were analyzed, within and across, for patterns involving students' solution strategies for the Fair Task. When patterns among student strategies were found, further analysis on the participants work exhibiting those patterns was conducted to determine the level of success among the strategies used. This completed the analysis for RQ2. Data excerpts to support the patterns are shared.

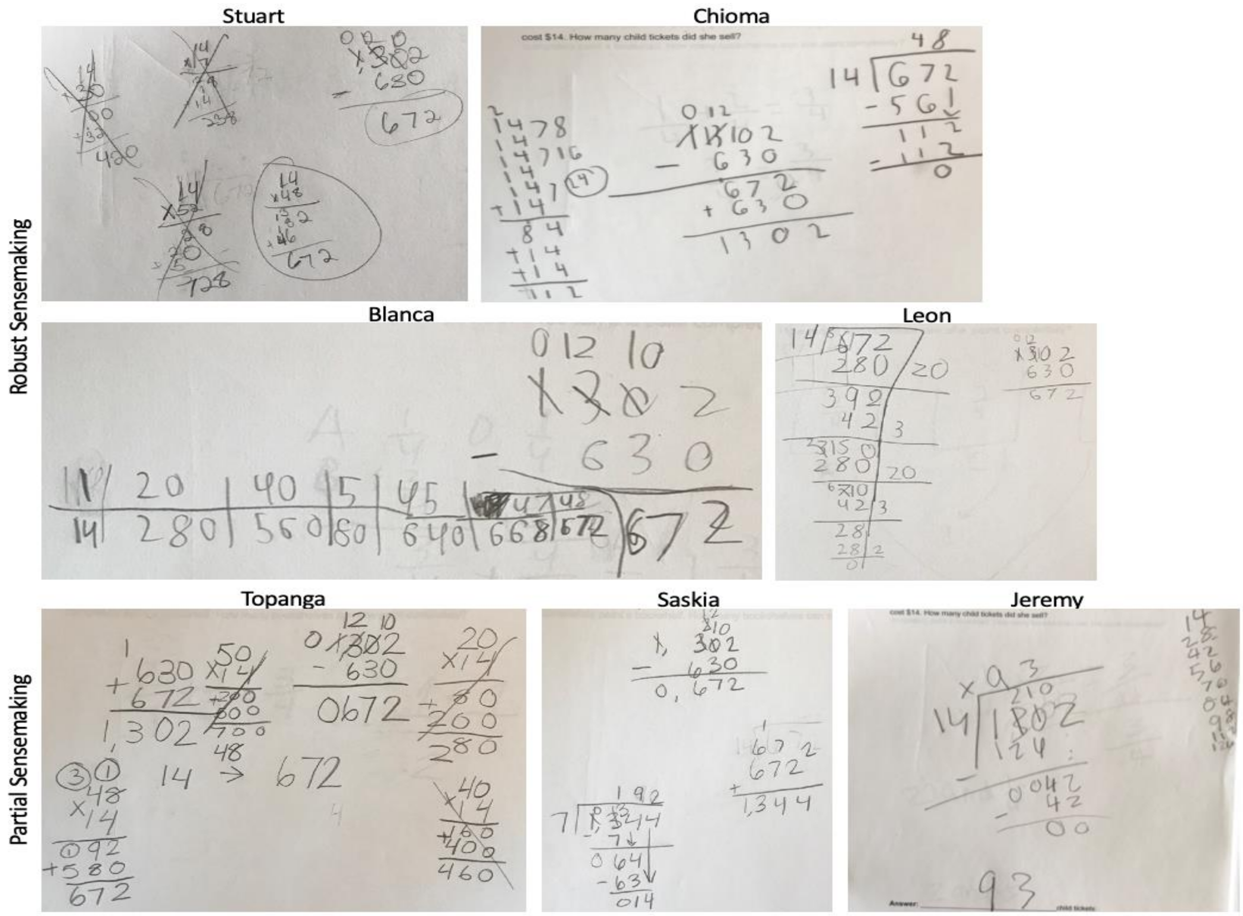
Findings

RQ1: Sensemaking of the Fair Task

Inductive analysis revealed three qualitatively different levels of student sensemaking. These domains were labelled as: robust evidence of sensemaking, partial evidence of sensemaking, and no evidence of sensemaking. *Robust evidence of sensemaking* about the Fair Task indicated attention to all three key observations necessary to solve the problem. Seventy-nine of the 280 students (28%) in our sample provided evidence that they made sense of the key observations and enacted 14 unique mathematical strategies to derive an answer. While strategies varied among the 79 students, 50 students arrived at the correct answer. The remaining 29 students had evidence of their sensemaking about all three KOs, but didn't arrive at the solution due to a minor arithmetic error. This suggests that generally speaking, students who made sense of the difference, the number of groups, and the number of groups within the appropriate difference, arrived at the appropriate solution. Figure 1 offers four samples of student work evidencing robust sensemaking through different strategies.

Figure 1

Student Samples for Robust and Partial Sensemaking



Note. Student work samples of different strategies for robust and partial sensemaking of the Key Observations needed to solve the Fair Task.

Some students in our sample demonstrated *partial evidence of sensemaking* about the Fair Task through their attention to mathematical work for KO1, KO2, or both KO1 and KO2, but did not provide evidence for KO3. One hundred fifteen of the 280 students (41%) provided evidence that they made sense of either the difference, the number of groups of 14, or both. However, these students were unable to demonstrate evidence of their understanding for KO3. This is depicted in the examples in Figure 1. The students in this domain exhibited eight mathematically different strategies.

Students' work lacking evidence for any of the three key observations were classified as *no evidence of sensemaking*. Eighty-six of the 280 students (31%) provided no evidence that they had made sense of any of the three key observations. Broadly speaking, students in this domain either enacted strategies that did not lead to a correct solution of the Fair Task or gave no evidence of how they arrived at their solution.

RQ2: Strategic Choices for Finding the Difference (KO1)

As students made sense of KO1 involving the difference between 1302 and 630, they had representational and operational choices to make. Three strategies were identified: (a) Standard Algorithm, which involves a symbolic representation to perform vertical subtraction (b) Adding Up, which involves a symbolic representation of adding up from 630 to arrive at 1302; and (c) Number Line, which involves a pictorial representation of adding up from 630 to arrive at 1302 using a number line. Standard Algorithm was the most prevalent strategy among students as it was used by 159 of 174 students who attended to KO1. Adding Up from 630, a much less prevalent strategy than Standard Algorithm, was used by 14 of 174 students who attended to KO1. Number Line was only used by one student.

RQ2: Strategic Choices for Finding the Number of Groups (KO2)

Students used a variety of methods to find the number of groups of 14 to represent the number of child tickets sold (KO2). Students used both number-based and digit-based operational procedures. The number-based operational procedures that students used included the following: repeated subtraction and multiplication, multiplication using the standard multiplication algorithm or the box method, partial quotients using a traditional or nontraditional setup, and compensation. The digit-based operational procedures that students used included the standard algorithm for division alone or in combination with repeated subtraction, addition, and/or multiplication. Table 1 shows the number of participants in each group including the number of participants who found the correct number of groups of 14 that go into 672. The evidence indicates that students who engaged in the Fair Task were most successful with finding the number of groups when using multiplication. Students were equally likely to be successful when using the standard algorithm and partial quotients to do traditional division and students were only successful 11% of the time when using repeating addition and subtraction.

Table 1

Student Strategies Attending to Key Observation 2

Strategy	Number Based or Digit Based	Total Participants	Number of participants with correct computation	Percentage
Repeated addition and subtraction	Number Based	9	1	11.11%
Multiplication	Number Based	10	10	100%
Standard Division Algorithm	Digit Based	39	26	66.67%
Partial quotients	Number Based	20	12	60%

Summary of Findings

The number of different strategic choices made by students demonstrated the open nature of the Fair Task. The various strategies illuminate differences in students' sensemaking and response to multi-step word problems. Out of the 174 students who gave evidence for making sense of KO1, 159 of them used the standard algorithm for subtraction, including all of the students who showed robust evidence of sensemaking. In contrast, KO2 opens up the pathway for division to be used as the students need to find the number of groups of 14 that go into 672. However, the students in this study used all four operations to make sense of and solve KO2. Students' strategies when sensemaking about KO2 showed that many understood they were able to use properties of operations and the relationships between the operations in their quest to find the number of groups. However, there were differences in the rates of success among the strategies as only two-thirds of the students using algorithmic processes for division proceeded to get the correct answer while 100% of the students using multiplication methods to arrive at the number of groups of 14 arrived at the correct answer. Lastly, only 28% of all students were able to demonstrate sensemaking about the connection between KO1 and KO2 and this greatly restricted the number of students who could successfully solve the problem.

Connections to Literature

The findings here support and extend the current sensemaking literature by examining students' sensemaking in each key observation of a multi-step word problem. Students struggled the most with KO3 (difference and number of groups of 14), which required them to makes sense how the two mathematical ideas connected. This supports Quintero's (1983) assertion about word problems difficulty and Pape's (2004) conclusion that sensemaking is essential for successful problem solving. This study also extends the problem-solving literature (e.g., Palm, 2008; Yee & Bostic, 2014) by providing evidence about which strategies chosen by fourth-grade students tended to yield the most success. Overall, sensemaking and procedural proficiency were revealed to be co-dependent attributes for fourth-graders successful problem solving.

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