Leveraging Modeling with Mathematics-focused Instruction to Promote Other Standards for Mathematical Practice

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Abstract

The Standards for Mathematical Practice (SMPs) describe mathematical proficiency in terms of behaviors and habits that every student should develop during mathematics instruction. Modeling with mathematics supports students in gaining facility with multiple representations and making sense of real-world phenomena. We investigated K-10 mathematics teaching for possible associations between mathematics teaching behaviors promoting modeling with mathematics (SMP4) and those identified by the other SMPs, using a classroom observation protocol called the Revised SMPs Look-for Protocol. Data consisted of lessons and videos of mathematics instruction from 70 K-10 mathematics teachers engaged in professional development focused on the SMPs. Results illuminated several associations between modeling with mathematics (SMP4) and other SMPs. A typical instructional case illustrates these associations and suggests potential for further opportunities. Teachers aiming to foster students' mathematical proficiency might consider instruction promoting SMP4 as a means to promote further connections to other mathematical behaviors and habits described in the SMPs.

Introduction

> *"School mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts outside of mathematics"* (National Council of Teachers of Mathematics, 2000, pp. 65-66).

Such opportunities likely involve modeling with mathematics, one of eight Standards for Mathematical Practice (SMPs) described in the Common Core State Standards (see Table 1; Common Core State Standards Initiative [CCSSI], 2010). This standard offers students the opportunity to connect real-life or lived experiences with mathematical problems presented in the classroom (Bostic, 2012/2013, 2015; Usiskin, 2015; Zawojewski, 2010). SMP4 also supports using multiple representations as a means to explain phenomena in everyday and mathematical terms (Bostic, 2015; CCSSI, 2010; Lesh & Zawojewski, 2007; Thomas & Bostic, 2015). In fact, SMP4 is the only standard that explicitly links classroom-based mathematics with the real world. It is uniquely positioned to foster connections among mathematical domains and traverse the division between classroom learning and everyday life. Thus, teachers should promote modeling with mathematics as a way to deepen students' mathematics knowledge (Bostic, 2015; Thomas & Bostic, 2015) and connect in- and out-of-class experiences through a mathematical lens (Matney, Jackson, & Bostic, 2013; Thomas & Bostic, 2015).

Table 1: Standards for Mathematical Practice

Standard for Mathematical Practice #	Title
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for regularity in repeated reasoning.

Note: Discussion about a specific SMP is denoted as SMP# within the manuscript.

There are instructional connections across the SMPs (Fennell, Kobett, & Wray, 2013; Kanold & Larson, 2012; Koestler, Felton, Bieda, & Otten, 2013). Some have argued that SMP1 and SMP6 are a connecting thread across the other SMPs (Fennell et al., 2013; Kanold & Larson, 2012). Engaging students in SMP1or SMP6 might be associated with fostering other SMP-like behaviors but there is no research-based evidence supporting that this will happen (Koestler et al., 2013). Conversely, SMP4 has unique features that may correlate with other SMP-like behaviors. That is, SMP4 might be a trigger mechanism that activates engagement in other SMPs because of its unique features, which include: a focus on a mathematical model; interactions between mathematical and nonmathematical (i.e., situational) referents; problems stemming from the real-world, particularly issues found in students' communities or the workplace; re-usability of mathematical models in other situations; and ideas communicated through oral and written language that make sense to the reader and problem solver (Bostic, 2015; Bostic, Matney, & Sondergeld, 2016; Floro & Bostic, in press; Thomas & Bostic, 2015). We conjectured that tasks promoting these features of SMP4 might associate with behaviors and habits found in the other seven SMPs. For instance, SMP4-focused instruction tends to foster opportunities for students to look for some underlying mathematical structure within a problem (Bleiler, Baxter, Stephens, & Barlow, 2015; Floro & Bostic, in press; Usiskin, 2015).

There is some evidence that promoting SMP4 supports other mathematical behaviors like those described in the SMPs (Floro & Bostic, in press; Thomas & Bostic, 2015); however, these studies and others often draw upon small samples (e.g., one or two teachers). There is little, if any, research-based evidence drawn from a larger sample of teachers' classroom practices exploring what SMP-related behaviors are also fostered when teachers promote SMP4 during classroom instruction.

The purpose of this study was to explore those correlational associations between SMP4 and other mathematical behaviors and habits described in the SMPs. We hypothesized that promoting SMP4 offers fruitful potential for encouraging other mathematical behaviors and habits described by the SMPs. Quantitative results and a classroom example are shared as an illustration to inform K-10 teachers' instructional practices as well as the decisions of mathematics teacher educators and professional developers. We used a mixed-methods approach to investigate possible connections between SMP4 and other SMPs and contextualize the correlations by giving instances from one teacher's classroom practice.

Context for Exploring K-10 Mathematics Instruction

Context and Participants

One hundred thirty-eight teachers located in a Midwest state volunteered to participate in one of two professional development (PD) projects. Projects met in separate locations due to geographic constraints. One project included K-5 mathematics teachers while the other was composed of grades 6-10 teachers. A shared goal of the yearlong PD projects was to foster teachers' understanding of the SMPs, particularly SMP4. An evaluation component within the projects included collecting and examining teachers' written lessons and instruction developed after experiencing the PD. Teachers were told to submit two lessons and video of them teaching one of those lessons. Instruction did not necessarily need to focus on promoting SMP4. In total, 70 grades K-10 teachers intended to promote students' mathematics proficiency through engagement in SMP4 during their videotaped lesson. Thus, these 70 teachers were a purposefully selected sample from a greater sample of PD participants. Our sample consisted of 29 grades K-3 teachers (early childhood), 35 grades 4-8 teachers (middle grades), and 6 grades 9-10 teachers. There were

16 male and 54 female teachers. On average, teachers had 13 years of teaching experience.

Instrument

Recent work by Fennell and colleagues (2013) led to a look-for protocol used by teachers, teacher educators, and mathematics supervisors. This protocol allows an observer to *look for* observable mathematics teaching behaviors that are related to the SMPs. This observation tool, used over 1,000 times in several districts, allows supervisors and teacher educators to create evidence-based records of teachers' instruction (Fennell et al., 2013). Our team revised the Fennell et al. (2013) protocol to create the *Revised SMPs Look-for Protocol* (Bostic et al., 2016). Those interested in a discussion of these revisions and the validation of the Protocol should reference Bostic et al. (2016). The revisions allow for a greater number of teacher-initiated moves to count as evidence related to an indicator. Appendix A shows the *Revised SMPs Look-for Protocol*, which includes descriptions for mathematics teaching behaviors related to the SMPs. For instance, one addition found in the revision was the phrase "and/or strategies" for indicator 1b. Content experts (i.e., mathematics teachers, supervisors, curriculum coordinators, mathematicians, and mathematics teacher educators) reviewed the *Revised SMPs Look-for Protocol* and expressed that it appropriately captured possible teacher moves indicative of promoting the SMPs.

Data Analysis

We analyzed our quantitative data in three phases. The first phase involved becoming familiar with the instruction. We read each lesson then watched the accompanying video in its entirety. The second phase was coding teachers' instruction seen in the videos using the *Revised SMPs Look-for Protocol*. A lesson received a score for each SMP based on the total number of indicators observed during the video. A score expressed the number of indicators per SMP. Thus, every lesson had eight values, one for each SMP. For example, a score of two for SMP4 meant that two indicators for SMP4 were observed on at least one occasion. Numerous instances of the same indicator for a SMP were coded the same as a single instance of an indicator for a SMP (i.e., 1). Teachers did not have the *Revised SMPs Look-for Protocol* prior to submitting their lessons and videos. Inter-rater agreement was high across coders (93%), which exceeded the minimum threshold (90%) needed to conduct quantitative analysis (James, Demaree, & Wolf, 1993). The third and final phase of our quantitative data analysis was conducting correlational analysis using these scores. Correlations such as Pearson's *r* are a measure of the strength of association between two variables (Shavelson, 1996). Statistically significant correlations for these data indicated that there was a genuine relationship between two SMPs, and there was a less than 5% likelihood that this correlation might happen by chance. All statistically significant correlations were interpreted using Cohen's (1998) guidelines: [0.01, 0.2] were considered weakly correlated, [0.21, 0.4] were moderately correlated, and [0.41,1] were strongly correlated.

We employed qualitative methods using inductive analysis (Hatch, 2002) to give meaning to the correlations. We selected Mrs. Gaston (pseudonym) from the sample of 70 teachers because her case was typical of the sample. Her case reified the numerous ways SMP4 and other SMPs appeared to be connected during classroom instruction. Inductive analysis allowed us to express salient connections that gave meaning to the quantitative results (Glaser & Strauss, 1967/2012; Hatch, 2002). Our approach to inductive analysis started with re-watching her video and reviewing her lesson. Step two was to make memos consisting of initial ideas stemming from the video and reflecting on observed indicators. Step three was to reflect on those memos and indicators as a way to synthesize them into key impressions. Step four was to search for evidence within her case to support our key impressions. Step five was to search the data for counter evidence within her case. Impressions with a paucity of counter evidence and a large set of evidence were retained. The sixth and final step was crafting clearly written impressions (themes) to share broadly that illuminate the connections between SMP4 and the other SMPs.

Results

In this section, we present descriptive statistics and correlations between teachers' promotion of SMP4 and other SMPs and then share a characterization of these correlations with descriptions from Mrs. Gaston's classroom instruction.

Descriptive Statistics

Means and standard deviations for teachers' promotion of the SMPs indicate that on average, teachers promoted numerous SMPs (see Table 2 on pg. 24). The most commonly seen SMPs were SMP3 and SMP5 whereas the least frequently observed SMPs were SMP7 and SMP8. These descriptive statistics demonstrate a picture of teachers'

Table 2: Descriptive Statistics for Teachers' Promotion of the SMPs during Instruction

	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8
Mean	1.21	1.05	1.44	1.93	1.46	0.8	0.47	0.56
SD	0.97	0.86	1.03	0.8	0.83	0.66	0.69	0.67

instruction that included numerous features of SMP-focused instruction.

Correlations

Results illuminated that teachers who promoted SMP4 tended to also foster other SMPs (see Table 3). First, there were several moderate correlations. SMP4 was moderately correlated with SMP1, SMP6, and SMP8. Moderate correlations suggested a greater positive relationship between SMP4 and other SMPs compared to weak correlations. Second, there was a weak correlation between SMP4 and SMP2 and SMP7. Weak correlations indicated some (i.e., more than none) relationship between SMP4 and another SMP. Finally, there was no statistically significant correlation between SMP4 and SMP5.

students on the memorization of known mathematical definitions, tricks, and processes to solve well-defined textbook and test-based problems. Mrs. Gaston's instruction seen on the video was fairly typical within the set of teachers' instruction we viewed. We purposefully selected her case (i.e., observed indicators) to share because her case was typical across the sample. Mrs. Gaston selected a ratio task that involved creating a drink mixture made from lemon concentrate and water. During the development of this lesson, Mrs. Gaston indicated that she wanted to encourage students' thinking about the situational and mathematical contexts within the topic of ratios. We describe her instruction, highlighting instances of when and how she promoted SMPs.

Table 3: Results from Correlational Analysis of K-10 Teachers' Instruction

Modeling SMP	All other Standards for Mathematical Practice							
	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8
SMP4	.27*	.18*	.23*	.14	.22*	.19*	.29*	0.56

* $p < .05$

Note: SMP4 was correlated with six of the remaining seven SMPs. All correlations are Pearson r values.

In sum, there was a good chance that teachers in this sample who promoted SMP4 were associated with fostering mathematical behaviors and habits found in at least one and possibly up to six additional SMPs. A challenge of interpreting these correlations was expressing what this looks or sounds like in the activities of a mathematics classroom. To address that challenge, we used inductive analysis to draw out some instances that illustrated how one teacher enacted SMP4-focused instruction and encouraged promotion of other SMP-related indicators.

A Case of Classroom Instruction: Mrs. Gaston

Mrs. Gaston was a seventh-grade teacher who had taught for 18 years in a small Midwestern school district. She described her previous instruction as lecture based, focusing

Nurturing classroom norms. Mrs. Gaston provided a learning space for her students to apply what they knew, comfortably make assumptions, identify important quantities, and map relationships using pictures, symbols, graphs, tables, and physical tools (SMP4) by nurturing classroom norms. Mrs. Gaston promoted students' involvement in SMP4 and used it as a lever to engage her students in SMP5. She reminded students of their usual classroom norms supporting SMP4.

> You are going to use pictures, props, tables, symbols, numbers, manipulatives and oh we're going to talk about it. You are not going to give up. If you find a way [to model and solve the task], guess what, I'm going to tell you, find another way.

Mrs. Gaston provided several tools students might use and she wanted them to strategically use whatever they needed to make sense of the problem. Thus, students considered the available tools, including representations, and selected them according to their own strategic competence (SMP5). In seeking to establish a learning environment conducive for SMP4, Mrs. Gaston focused students throughout the lesson on her desire for them to "prove it with a picture" and encouraged them to persevere in finding multiple strategies.

Mrs. Gaston drew on the SMP4-focused instruction as an opportunity to also activate students' engagement in SMP3. A classroom norm she fostered was respect for peers' ideas. She said to the class:

> You know as you participate you're going to listen to each other. You're going to give each other your attention. So when you're working in your groups don't ignore people. . . . You're going to listen by not speaking when someone else is giving their ideas. If you do not agree with someone in your group you're going to ask questions: What do you mean? What are you doing?

Mrs. Gaston reminded students to listen to one another and ask questions as a means to foster peers' model development.

Lesson launch. During the lesson launch, Mrs. Gaston offered her students an experience in a context similar to the day's task. She enacted the process of making lemonade from frozen concentrate. She asked students if they had made lemonade this way previously and most raised their hands and/or shouted, "Yeah!" Then, students proceeded to tell Mrs. Gaston how to make it. As instructed by her students, Mrs. Gaston opened three cans of frozen lemonade and dumped the contents into a large clear container. In the following dialogue, Mrs. Gaston capitalized on her instruction promoting SMP4 as a means to concomitantly engage her students in attending to precision (SMP6). She filled a separate large container with water then the following dialogue occurred.

> **S1:** Then you put the water in.
>
> **T:** Then I put the water in?
>
> S1: You need the [lemonade] container to pour it into the [large mixing] container. Because you need three of those little [lemonade] containers filled with water in it [the large mixing container].
>
> T: Ok, so you mean I can't just go like this? (Motioning to pour the whole pitcher of water in the container without measuring.)
>
> S2: Well you can.
>
> T: (pouring very little water into the container.) What if I stop now? [PAUSE]
>
> S2: You don't know what the ratio is.
>
> T: Cause what?
>
> S2: You don't know what the ratio is.
>
> T: Oh, I don't know what the ratio [emphasis added] is? The ratio of what to what?
>
> S3: The ratio of the lemonade in the container to the water in the container.
>
> T: So I need to know the ratio? Cause what if I stop right now? What would this taste like?

In this instance, Mrs. Gaston emphasized the word ratio as a means to highlight the importance of mathematical vocabulary connected to the situational context of the problem (SMP6) during this dialogue. Mrs. Gaston planned for this in her lesson; she drew attention to students' academic language use and intended to foster their precision with mathematical vocabulary, in this case with the word *ratio*. First, Mrs. Gaston started by asking the student to say it again. Then, she changed the tone of her voice to emphasize the word ratio. Later, the student mentioned ratio but did not go further to explain what two things were distinguished in the ratio relationship. Mrs. Gaston asked a question to help the student more precisely use the mathematical language of ratio within the situational context.

The class proceeded to comment on how too little water makes the lemonade strong and too much water makes it weak. They indicated that the ratio is important to make it just right. Thus, students were engaged in discussing a situational model of ratios before proceeding to mathematically model this ratio in the focal task. Later, the class identified contexts similar to this one such as making cookie dough, scrambled eggs, pumpkin pie, and cinnamon rolls. Mrs. Gaston shared a story with the class about a family member who made a pumpkin pie that tasted unusual because the individual who made the pie confused the ratios for sugar and salt hence the pie was salty rather than sweet. Then, students expressed other similar scenarios where a recipe was not followed correctly and the importance of understanding how different ratios apply to different contexts; hence, they perceived the importance of transferring ideas from this situational model to other contexts.

The lesson included numerous opportunities to engage students in SMP4 as well as an opportunity for students' engagement in SMP1. When Mrs. Gaston focused on SMP4 as she did in this lesson, she intentionally planned a lesson drawing on a relatable context that in turn allowed for more entry points into the task. This intentional focus on SMP4 brought about students' engagement in SMP1. During the lesson launch, the students reflected on their past experiences with analogous contexts and connected the launch to their own lives. This launch provided a foundation from which students could make sense of the focal task and persevere in constructing and sharing viable mathematical models to explain their thinking.

Focal task. The focal task stemmed from adapting a Connected Mathematics Project 2 task (Figure 1; Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006).

Mrs. Gaston's lesson plan indicated that she had a two-fold instructional goal for students: to answer the problem and to develop a mathematical model for judging the strength of other mixtures in different contexts. Students were given approximately 40 minutes to reflect on the problem, discuss it with a partner, and construct a brief presentation about the models for solving the problem.

Mrs. Gaston reconvened the class for presentations, which included discussing various models that demonstrated which solution had the strongest concentration of lemon flavoring. Fostering students' engagement in SMP4 during this instructional moment supported students' mathematics learning through SMP2-related behaviors. That is, behaviors indicative of SMP2 included a focus on decontextualizing and contextualizing from a mathematical problem. One group shared a bar model approach to determine the

FIGURE 1.

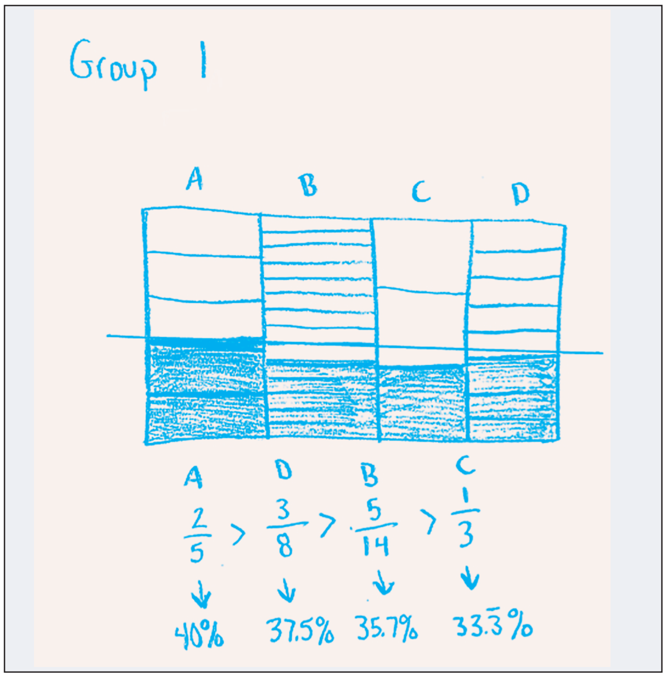
This was the focal task for Mrs. Gaston's instruction intending to promote modeling with mathematics.

Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times. One morning, Julia and Mariah make lemonade for all the campers. They plan to make the juice by mixing water and frozen lemon juice concentrate. To find that mix that tastes best, they decide to test some mixes.

Which mix did they decide tastes the most lemony?

percentage of lemon concentration that was found in the total amount of the solution (see Figure 2). Another group used a similar model (see Figure 3), and found percentages to compare how much lemon flavoring there was to how much water using circles. However, they modeled different ratios from the first group. A third model from another group involved comparing solutions using common denominators (see Figure 4).

FIGURE 2.
One group's approach for solving the focal task used a bar model representation. The horizontal line showed that mixture A was the "most lemony." The shaded region represented the concentrate and the unshaded region represented the water in the mixture. The percentages represented the ratio of concentrate:total mixture contents.



Throughout the whole-class discussion and presentations, students asked peers to explain what they meant, as seen in one interaction during the discussion of the first model (see Figure 2). We share an excerpt from the discussion, which in this case is between a peer (S4) who asked questions to one of the presenters (S5).

S4: What were the percents? And like what were the percents showing?

S5: Um, which one is greater. Because forty would be greater than thirty-seven point five, thirty-three point three three three and thirty-five percent so it's greater than all of them.

FIGURE 3.
A second group's approach used circular models to solve the focal task. The shaded region represented the amount of concentrate in the mixture. The unshaded region represented the amount of water in the mixture. The percentages represented the ratio of concentrate:water.

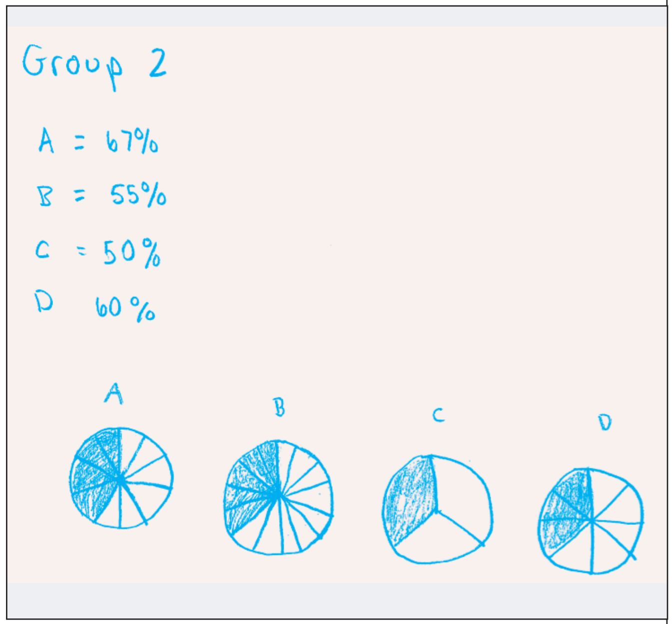
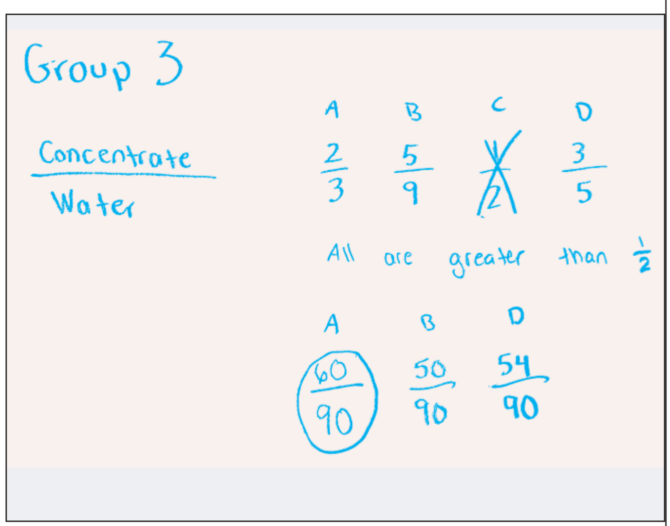


FIGURE 4.
A third group's model used common denominators for solving the focal task. Mixture C was eliminated because students immediately recognized all other fractions were bigger than ½. Fractions with common denominators represented the ratio of concentrate:water.



S4: Wouldn't you have to, um, measure something about the amount of water because the water could be like higher in all the others and could even out?

S5: Well out of one hundred percent, forty percent is greater, like one glass is a hundred percent, so this would be greater than thirty-five percent because forty percent is greater out of a hundred percent. So percent wise, it's higher.

Students sought to make sense of one another's models and justify the mixture that had the strongest lemon flavor. This prompted opportunities to discuss and use representations to make sense of quantities and their relationships as well as opportunities to decontextualize and contextualize (SMP2). The above dialogue also provides an example of a student (S4) questioning another student (S5) for the purpose of drawing out a more precise meaning of the language being used. Mrs. Gaston and her students often asked one another to tighten their language within justification statements and to precisely communicate the connection between the referents in pictures, quantities, and symbolic expressions such as the percent sign and inequalities (SMP6). For example, during Group 1's presentation of their model (see Figure 2) the following dialogue occurred.

T: Ok, I have, I have a question, Why did you, I'm looking at your picture.

S6: Yeah.

T: And then so, A) Why did you divide all of your pictures, in like differently, like A spaces are larger than B spaces in between those little lines?

S6: Because these are each, I probably should have um, made them, all have a common denominator but, they each are, they're each different fractions.

T: Ok. So then you, ok, then I like the way you lined it up like that. So, you could have had a common denominator then?

S6: Yeah, but they probably should all go together so.

In this case, Mrs. Gaston asked the presenting student (S6) to explain the dividing lines in the group's representation (see Figure 2). The question prompted the student to consider something new (common denominators) that could have made the relationship among the ratios and representation more clear. Through these kinds of classroom

Table 4: Mrs. Gaston's Case: Connecting SMP4 Focused Instruction to other SMPs

SMP 4 Teacher Indicators	Mrs. Gaston's Teaching Behaviors	SMP Look-for Protocol Indicators
A. Use mathematical models appropriate for the focus of the lesson	In launching the lesson, the teacher used a sensible real-world context that was familiar to the students and directly connected to the day's mathematical task, which could be approached through multiple strategies.	SMP 1A & SMP 1B
B. Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids)	To promote model development, the teacher nurtured norms promoting the use of manipulatives. During individual and whole class discussion, the teacher encouraged precise mathematical language as students articulated why their models were appropriate.	SMP 5A & SMP 5C SMP 6A & SMP 6C
C. Remind students that a mathematical model used to represent a problem's solution is 'a work in progress,' and may be revised as needed	To promote model development, the teacher nurtured the norm of respecting others' ideas by considering and listening to one another. She reminded students to ask questions as a means to foster peers' model development. During individual and whole class discussion, the teacher encouraged precise mathematical language in student articulation of ideas.	SMP 3A & SMP 3C SMP 6A & SMP 6C
D. Employ problems arising from everyday life, the local community, society, and workplace such that the solution is a model to reuse.	The teacher launched the lesson with a context connected to students' everyday lives. The planned task for students to consider involved everyday life situations in which the students must develop a mathematical model as part of the solution. The teacher leveraged these everyday life contexts and a solution model to promote students contextualizing and decontextualizing.	SMP 2A, SMP 2C1, & SMP 2C2

interactions, Mrs. Gaston and the students made moves to improve their understanding of one another's mathematical ideas and the precision of the use of mathematical language, representation, and referents (SMP6).

Mrs. Gaston aimed to promote SMP4-focused instruction, which also happened to offer opportunities for her students' to engage in SMPs 1, 2, 3, 4, 5, and 6. The case of Mrs. Gaston shows what is possible when a teacher works to promote SMP4 and gives a qualitative picture of how the correlations from the quantitative Look-for Protocol occurred in the teachers' classroom practice. Table 4 consolidates the case of Mrs. Gaston to show the connections between the codes for SMP4 and the other SMPs found during her instruction.

Implications for Mathematics Teaching

When considering this instance and several others as a teacher's first foray into instruction promoting SMP4, there are quite a few wonderful developments within the instructional scenario. One example is students developing and defending their models and justifications. A second example is the observable evidence of student engagement in multiple SMPs during the lesson. We are encouraged by these results, both the quantitative and the qualitative. As evidenced by the correlations in this study, teachers who focused their instruction on fostering SMP4 also demonstrated that their instruction facilitated opportunities for students to engage in multiple SMPs.

Although some have argued that SMP1 and SMP6 connect with the other SMPs (Fennell et al., 2013, Koestler et al., 2013), there is no guarantee that promoting SMP1 or SMP6 will always foster SMP4 much less other SMPs. However, we can conclude from our analysis that instruction by teachers in this sample who intended to promote SMP4 had a reasonable chance of also encouraging SMP1, and a slightly lesser chance for encouraging SMP6. Relatedly, K-10 teachers' instruction promoting SMP4 connected with reasoning abstractly and quantitatively (SMP2), constructing viable arguments and critiquing others' reasoning (SMP3), and looking for and expressing regularity in repeated reasoning (SMP8). Though the correlation between SMP4 and SMP5 was not statistically significant across our sample, Mrs. Gaston's instruction showed that SMP5 was not wholly absent from SMP4-focused instruction. We conclude that when teachers in our sample focused on promoting SMP4, it provided natural opportunities to foster engagement in other SMPs during mathematics instruction. These SMP connections may allow students to make sense of mathematics at a deeper level by building conceptual understanding and effectively linking mathematics learned in school with real-life experiences (Bostic, 2012/2013, 2015; Matney et al., 2013; Thomas & Bostic, 2015). In sum, our conclusion is that instruction promoting SMP4 has the propensity to support engagement in other mathematical behaviors and habits described in the SMPs. SMP4-focused instruction offers opportunities for students to engage in mathematics within tasks drawn from relevant contexts and connect ideas among various situational contexts.

Implications for Mathematics Teacher Educators and PD Providers

The ideas in this manuscript stemmed from working intensely alongside teachers to help them grow in their understanding of mathematical behaviors and habits described in the SMPs, which assisted their ability to design and enact instruction promoting the behaviors and habits. Many mathematics teacher educators are enacting PD for mathematics teachers around the SMPs with an aim to understand them and make them a part of regular instruction. An implication of our research is that fostering mathematics teacher's understanding of SMP4 and concomitantly their abilities to design SMP4-focused instruction may be fruitful for promoting other SMPs. It may be a good idea for mathematics teacher educators and PD providers to initiate mathematics teachers' thinking about the SMPs by starting with developing a deep understanding of SMP4 then following up with the other SMPs.

This manuscript also provides mathematics teacher educators and PD providers with a real-life scenario of how one teacher promoted SMP4 as well as several other SMPs. Mrs. Gaston's lesson might ignite and foster discussions about how SMP4-focused instruction leveraged other SMPs to also appear during the same lesson. Discussions, along with unpacking the correlational results, may spur thinking about possible connections between SMP4-focused instruction and other SMPs. As a reminder, these teachers' promotion of other SMPs is correlated with, not predicted or caused by, SMP4.

Further Questions: SMP4 and Predictive Validity

We aimed to illustrate correlations between SMP4-focused instruction and other SMPs in this mixed-methods study. The focus of this study was exploring correlational relationships; however, we cannot provide evidence about causal or predictive relationships. Correlations suggest the likelihood of two outcomes occurring and are often conducted before causal or predictive studies (Shavelson, 1996). Causal and predictive studies use ANOVA or regression as a means to explore whether one outcome is caused or predicted by the occurrence of another outcome (Shavelson, 1996). An experimental design could illuminate such potential causal relationships between SMP4 and other SMPs. One such design might include 30 teachers enacting the same lesson, which includes a strong SMP4 focus, to their students in their typical learning environments. The independent variable in this case might be presence of teachers' promotion of SMP4-like behaviors and dependent variable might be presence of teachers' promotion of other SMP-like behaviors. Analyses of teachers' promotion of SMP-like behaviors, using a logistic regression might illuminate any causal relationships. At this time, we cannot make any predictive statements suggesting that promoting SMP4 causes other SMPs to be promoted. We hope future research might take up this call for a causal or predictive study employing a methodology like this one described here or otherwise. Results from the present study allow us to conclude that teachers' promotion of SMP4-related behaviors is related (i.e., occurring within the lesson) to teachers' promotion of several other SMP-related behaviors. ✪

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APPENDIX A.

Revised SMPs Look-for Protocol

Place a mark in the box next to the appropriate indicator when observed.

Mathematical Practices	Teachers
1. Make sense of problems and persevere in solving them	☐ A. Involve students in rich problem-based tasks that encourage them to persevere in order to reach a solution. ☐ B. Provide opportunities for students to solve problems that have multiple solutions and/or strategies. ☐ C. Encourage students to represent their thinking while problem solving. *NOTE: Task must be a grade-level/developmentally appropriate problem. That is, a solution is not readily apparent, the solution pathway is not obvious, and more than one pathway is possible.* Comments:
2. Reason abstractly and quantitatively	☐ A. Facilitate opportunities for students to discuss representations or use representations to make sense of quantities and their relationships. ☐ B. Encourage the flexible use of properties of operations, tools, and solution strategies when solving problems. ☐ C1. Provide opportunities for students to decontextualize (abstract a situation) the mathematics within a mathematics task. ☐ C2. Provide opportunities for students to contextualize (identify referents for symbols involved) the mathematics within a mathematics task. *NOTE: Must have C1 and C2 to receive credit for indicator.* Comments:
3. Construct viable arguments and critique the reasoning of others	☐ A. Provide and orchestrate opportunities for students to listen to the solution strategies of others, discuss alternative strategies or solution(s), and defend their ideas. ☐ B. Ask higher-order questions which encourage students to defend their ideas, consider student(s) response(s) before making code. ☐ C. Provide prompts/tasks that encourage students to think critically about the mathematics they are learning, must be related to argumentation or proving events. ☐ D. Engage students in proving events that encourage students to develop and refine mathematical arguments (including conjectures) or proofs. Comments:
4. Model with mathematics	☐ A. Use mathematical models appropriate for the focus of the lesson. ☐ B. Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids). ☐ C. Remind students that a mathematical model used to represent a problem's solution is 'a work in progress,' and may be revised as needed. ☐ D. Employ problems arising from everyday life, the local community, society, and workplace such that the solution is a model to reuse. *NOTE: Must have D to be considered a task embedded within instruction promoting modeling with mathematics.* Comments:

Mathematical Practices	Teachers
5. Use appropriate tools strategically	☐ A. Use appropriate physical and/or digital tools to represent, explore, and deepen student understanding. ☐ B. Help students make sound decisions concerning the use of specific tools appropriate for the grade level and content focus of the lesson. ☐ C. Provide access to materials, models, tools, and/or technology-based resources that assist students in making conjectures necessary for solving problems. *(Students must use the resources.)* *NOTE: Representations do NOT count as tools.* Comments:
6. Attend to precision	☐ A. Emphasize the importance of precise communication by encouraging students to focus on clarity of the definitions, notation, and/or vocabulary used to convey their reasoning. ☐ B. Encourage accuracy and efficiency in computation and problem-based solutions, expressing numerical answers, data and/or measurements with a degree of precision appropriate for the context of the problem. ☐ C. Foster explanations and justifications using clearly articulated oral and/or written communication and grade-level appropriate conventions. Explanation or justification must go beyond Initiate-Respond-Evaluate (IRE.) Comments:
7. Look for and make use of structure	☐ A. Engage students in discussions emphasizing relationships between particular topics within a content domain or across content domains. ☐ B. Recognize that the quantitative relationships modeled by operations and their properties remain important regardless of the operational focus of a lesson. ☐ C. Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways e.g., 76 = (7 x 10) + 6; discussing types of quadrilaterals, etc. ☐ D. Encouraging examinations of a 'signal' and 'noise' in statistics-related tasks. Comments:
8. Look for express regularity in repeated reasoning	☐ A. Engage students in discussion related to repeated reasoning that may occur while executing a problem-solving strategy or in a problem's solution. ☐ B. Draw attention to the prerequisite steps necessary to consider when solving a problem. ☐ C. Urge students to continually evaluate the reasonableness of their results during problem solving. Comments: