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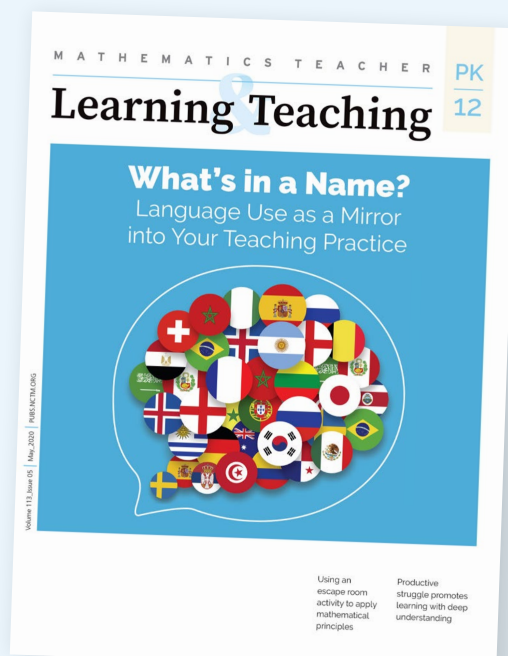

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Global Connections through Mathematical Problem Solving

Teachers from two countries designed a model-eliciting activity about the global issue of wind energy. They share teaching and student outcomes from a cross-border engagement in the task with students from Indonesia and the United States through synchronous video conference.

Rachel Wiemken, Russasmita Sri Padmi, and Gabriel Matney

As teachers of mathematics, we recognize that our world continues to become more connected. Nations of the world share concerns on many critical issues in such a way that communication and collaboration are important skills for the 21st century (Trilling and Fadel 2009). Hence, connecting global issues to our mathematics classroom is important. Through such learning experiences, students can appreciate that mathematics has a special place among the disciplines in forging understandings between people and nations as well as solving global problems. We enjoy connecting our students to real-world issues through open-ended problems in which they can invent and test their mathematical models (Lesh et al. 2000).

Real life poses problems that require selection of needed information, summoning and combining necessary skills and knowledge, transferring them to a real-life context, and developing a model

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that depends on initial assumptions. The fourth of the Common Core State Standards for Mathematical Practice (SMP 4) expresses the goal for students to proficiently model with mathematics:

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (NGA Center and CCSSO 2010, p. 7)

One benefit of modeling is that students experience the role of mathematics in human life. Providing students with real data and a realistic context that needs mathematization challenges their intellect and allows them to establish connections between mathematics and all sorts of human endeavors. In what follows, we share the experiences of secondary school students from Indonesia and the United States as they modeled with mathematics and justified their models to one another. We describe the Borean Winds task, its context within the larger global community, and how students progressed through each portion of the task, including students' thoughts and mathematical models. We provide readers with a rich task to implement in their classrooms. The task is an exemplar of the benefits of collaborating with a school in another country. Because of physical restraints concerning our health and safety with regard to COVID-19, this style of synchronous video conference collaboration demonstrates how to engage in rich, collaborative tasks without physically being next to each other.

BACKGROUND AND CLASSROOM CONTEXTS

Connecting students across borders has been an interest of each author. We have all previously traveled to several countries, which has expanded our educational experiences through conferences, school visits, conducting research, and more. Having personally experienced the educational benefits of a cross-culture collaboration, we wanted to find ways for our students to have these experiences as well. This project began when Gabriel Matney gave a keynote talk about globally connecting students through modeling at an Asia-Pacific Economic Cooperation meeting (Matney 2017). We sought to develop a globally relevant task about energy sustainability that required students to engage in mathematical modeling. Russasmita Sri Padmi reached out to school partners in Indonesia while Matney and Rachel Wiemken connected with school partners in the United States. We were able to find a teacher from each country to collaborate with to teach the lesson. For more information about school context and students, see table 1.

We met and coordinated with the two teachers on several occasions to create the lesson and work through the details of teaching the Borean Winds task. The time difference between the countries is 12 hours, so the project had synchronous and asynchronous aspects. The students from the United States agreed to come back to school from 8 p.m. to 9:30 p.m. on two separate occasions for interacting synchronously with their Indonesian peers. The classes met up synchronously through video communication technology to discuss their respective cultures and seek commonalities. Next, students from each country worked asynchronously through the task in small groups facilitated by their teacher. The classes had two 90-minute sessions to make their mathematical model. Finally, through screen sharing and video technology, students demonstrated and justified their models through tools such as

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spreadsheets, online graphing calculators, and presentation software.

BOREAN WIND SYSTEMS MODELING TASK

To launch the task, the teachers engaged the students in the following questions. Students were asked to think in small groups of three or four. The task is as follows:

> Borean Wind Systems is one of the leading companies in modern energy. This company is exclusively dedicated to wind energy. For what reason(s) would this company be so passionate about wind energy? What do you believe are the main goals of this company? How do you think they are going to achieve these goals?

We wanted to engage students in the scenario with the mindset of both a person associated with the Borean Wind Systems Company and a citizen of one of the countries involved. We carefully designed the questions so that they provided an entry point for all students. The US students came from communities where wind power is part of their local energy solution. All students have seen the wind turbines operating live near their school locations and have a basic understanding that wind is a natural resource converted to electric power. Indonesian students had also encountered wind turbines through their learning in science classes and through projects. Any of their life experiences or knowledge could be drawn on to consider the aforementioned questions. Students' answers included the following:

- "They want to reduce greenhouse gas emissions and use a renewable energy resource."
- "They want to make a profit. They know that renewable energy is trending and want to be on the forefront."
- "Wind turbines and their collection of energy are nonconsequential to our environment."

Following this discussion, the students were provided with more information on the Borean Wind Systems Company and the specific task to be solved.

> You have been hired by the Borean Wind Company. They have developed a new type of wind turbine that theoretically generates 20 percent more energy than the previous wind turbines. Borean Winds would like to test these new wind turbines in at least five different countries around the world. For this test, Borean Winds will donate 260 turbines across the selected countries. Several countries interested in receiving these wind turbines include Chile, China, Indonesia, Mexico, New Zealand, Russia, Thailand, and the United States. Each of these countries want to reduce their CO_2 emissions, and receiving these new wind turbines will help them do that.
>
> Your team is given the task to develop a method for fairly distributing the 260 wind turbines to at least five of these countries. What information would your team like to have to decide how to fairly distribute the 260 wind turbines?

The teachers asked the students to think carefully about the meaning of the phrase *fairly distributed*. Students discussed the idea of "fairness" in their small groups and shared in a whole-class discussion. At first, some students suggested that this meant everyone should get the same amount. Other students challenged this notion of fairness and insisted that reasons should be given for who gets the wind turbines, mentioning that "it wouldn't make sense to put wind turbines in a place where the wind rarely blows enough to spin it." In this part of the task, the students had to identify the information they thought was necessary to solve the problem (NGA Center and CCSSO 2010, p. 7), simultaneously considering the goals they mentioned at the start of the lesson.

Table 1 Information about School Context and Students Involved in the Project

School Location	Teacher Experience	No. of Students	Grade Levels	Subjects of Study during Project
Indonesia (urban)	23 years	15	11–12	Statistics
United States (rural)	3 years	15	9–12	Algebra, geometry, precalculus

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If the goal of the company was to make money, what information do they need to know about each country to be fair and still make a profit in the future? What if the goal is to reduce carbon dioxide (CO_2), or to harvest the abundantly available renewable energy sources? Balancing between the goal of the company and the need to be fair was a rich ground of discussion for students, where they could brainstorm different ideas and perspectives. We found that our students developed significant understandings of the mathematics role in human life through these considerations.

Furthermore, the task raised students' awareness about the complexity and meaning of fairness. Is it fair to equally distribute all the turbines? Is it fair to give some countries no turbines? Students used their prior knowledge and experiences to interpret their definition of fairness for this context while identifying what type of data would best help support their ideas. This task reverses the typical textbook mathematics problem. Instead of giving specified problems and providing the students with everything they need to reach the solution, the modeling task allows contextual agency by letting students define goals and conditions as well as identify relevant data by themselves. This higher-order thinking evokes meaningful conversation led by the students' ideas. Below are sample students' responses of data sets they felt they needed to reach a solution:

- Population
- Money a country earns
- Geography: wind map, size, location, topographical map
- Amount of pollution
- Greenhouse gas emissions

Next, the main task is posed to the students, along with a data sheet (see the appendix in the supplementary online materials) that they use to devise a plan. Because we required the students to use real-world data, we verified the veracity of information on the data sheet. This data sheet contained information from valid and reputable sources for students to use and included most of the items the students requested.

> ***Task:*** The company is able to provide the datasheet to help your team make a decision. On the basis of these data, your team needs to do the following:
>
> 1. You will create a presentation demonstrating how you reached your decision for distributing the wind turbines and a mathematical model that backs up your reasoning. Your justification is important because your company will use it when presenting who will receive the wind turbines.
> 2. In addition to your justification, you will attach the list of countries and the number of wind turbines you donated to each country.

Once the students received all the requirements for the task and made sense of the problem, they began engaging with the data sets, as listed in the appendix, to see how the information given was connected. They discussed different mathematical approaches they wanted to take to solve the problem, including what to do if the information they identified as necessary was not provided by the datasheet (see the appendix). The task provided students with opportunities to apply what they knew and make assumptions to simplify a complicated situation (NGA Center and CCSSO 2010, p. 7). For example, the real-world data for wind power capacity have no value for some countries (see the appendix). Students discussed different assumptions about why those countries have unknown values for wind power capacity and how to represent a null data point in their mathematical model.

US students pondered the idea of weighting each data category on importance felt by the group. Several groups made lists for each data category. For example, if they wanted to prioritize countries with a high carbon footprint, they would create a list of countries from high to low CO_2 emissions. They did this for several data sets and compared the lists to determine which countries should receive the wind turbines. They then used these lists to double-check their mathematical model at the end to see if it accurately represented the data in the way that they expected. Some groups considered weighting the different data sets that were provided and setting up some form of ratio. Because they examined different ways to approach the problem, they simultaneously considered the meaning of the problem and explored the parameters. For example, in situation A, one student questioned his group as to whether they wanted to give turbines to all countries because they technically needed to give out turbines to only five countries.

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Situation A:

US Student 1: Oh, that's another good question. Do we want to give it between the five countries, or should we spread it out among all the countries?
US Student 2: I think we should see how many we can give.
US Student 1: Because theoretically if we are doing it with percentages how we are, it should come out to . . . the combination of all the countries will equal 260.

This exchange led the students to discuss reasonable decisions based on data about why they would prioritize some countries over others.

The strategy most Indonesian students opted for was examining the proportion of each variable to the expected number of wind turbines. On the basis of their assumption in the beginning about determinant variables to the number of wind turbines, they investigated whether the relationship between the number of wind turbines and the variable was likely a direct or indirect proportion (see figure 1).

Situation B:

Indonesian Student 1: The wind power and GDP . . . country's income.
Indonesian Teacher: How does the country's income affect the number of wind turbines?
Indonesian Student 1: Well, some countries are able to buy turbines, and some can't; so those who can't [afford them] can get more turbines, and those who can afford it can get less.
Indonesian Teacher: So, if we have to talk in mathematics terms, we have this thing called *proportion*. What do you think is the relationship between the number of turbines and the income of a country?
Indonesian Student 2: The other way around? Like one goes up, and the other goes down.
Indonesian Student 1: That's indirect proportion.

The students realized that including all available data categories as variables in one formula is neither an efficient nor an effective strategy. They decided what variables to include and to omit. They multiplied the variables that were directly correlated with the number of wind turbines and divided by the variables with indirect proportion, resulting in a mathematical model (see figure 2).

Through the design of this task and all variables provided in the data sheet, we created an open-ended problem-solving space for multiple models to emerge from the students. They were free to attempt different variables and apply different mathematical concepts, as long as it gave them a reasonable model of the wind turbine distribution.

At this point in the lesson, the students were now creating and testing the initial models they created. The teachers facilitated conversations about the students' mathematical models to encourage rigorous

Fig. 1

Sample Student Models: Indonesia Model 2

$$X = \frac{\left(\frac{E}{P}\right) * G * W}{\Sigma\left(\left(\frac{E}{P}\right) * G * W\right)} * 260$$

Chile	0
China	24
Indonesia	9
Mexico	6
New Zealand	1
Russia	11
Thailand	0
United States	209

E=C0*$_2$ *emissions from each country
P= population count of each country
H= GDP of each country
W= wind speed
X= number of turbines received for each country

A sample of the second model presented by Indonesian students shows their mathematical operations, variable definitions, and a chart of how many wind turbines their model gives to each country.

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interpretation of the result in the context of the situation, reflect whether the solution was logical, and revise the model if it was not (NGA Center and CCSSO 2010, p. 7). This helped the students productively struggle through the difficulties they encountered, such as distributing more than 260 turbines, distributing fewer than 260 turbines, or generating an arbitrary number that lacked mathematical reasoning. The students demonstrated a desire to become experts on their model because they knew they would defend their reasoning to peers from two different countries. This required students to think precisely about all aspects of their model and the reasoning behind every mathematical decision, which in turn deepened their understanding.

PRESENTATION OF MATHEMATICAL MODELS

Students first presented their models to their in-country peers. The two best models in each country, as voted on by the class, would go on to be revised and justified to students of the other country. According to Koestler et al. (2013), the process of justification and revision helps students expand their mathematical repertoire, simultaneously reinforcing the idea that mathematical models are always provisional. After the students developed and refined their models, they created a multimedia presentation of their choice. From here, students from both countries engaged in a video conference. After the students made their case as to why their model was the fairest way to distribute the wind turbines, other students in the United States and Indonesia asked the presenting group questions and made comments about their mathematical model, as seen in situations C and D. In situation C, US students are presenting their model, which can be seen in figure 3.

Situation C:

Indonesian Student 3: I was wondering, where do you guys get the data of the wind power capacity based on unknown values?

US Student 3: So the unknown, we figured that if a country was not going to be able to—or willing to—get information on their previous wind turbines, they are not going to be as invested in wind turbines. Therefore, companies-wise thinking on that terms, it wouldn't be as profitable, so on that sort of half of the 50, we said that those were zeros because of that. If that makes sense to you?

Indonesian Student 3: Yes, yes.

Indonesian Teacher: OK, so for the countries that don't give any data on wind power, you decided not to give them any wind turbines? Is that right?

US Student 4: Uh, no, we decided that for the wind power capacity contribution, that part of the equation [*points*] up there, they got a zero. Because they were not willing to present the information to us.

In situation C, the students shared one of their model's main assumptions, which was necessitated by

Fig. 2

Sample Student Models: Indonesia Model 1

Step 1: $N = \frac{E * P}{H}$

Step 2: $X = \frac{N}{N_{total}} * 260$

E=C0$_2$ emissions from each country

P= population count of each country

H= GDP of each country

X= number of turbines received for each country

Chile	1
China	192
Indonesia	2
Mexico	25
New Zealand	1
Russia	17
Thailand	7
United States	17

When Indonesian students realized that including all available data categories as variables in one formula was inefficient and ineffective, they omitted some variables. They multiplied variables that directly correlated to the number of wind turbines and divided by the variables with indirect proportion.

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Fig. 3

Sample Student Models: US Model 2

$$\frac{9(A_1)+6.5(B_1)+5(C_1)}{\sum_1^8(9(A_n)+6.5(B_n)+5(C_n))}(260)= W_n$$

A=CO_2 emissions

B= Wind capacity

C= GDP per person

W_n= # of windmills for each country

Chile	1
China	125
Indonesia	7
Mexico	22
New Zealand	1
Russia	20
Thailand	4
United States	80

In a video conference, US students made a case as to why their model was the fairest way to distribute the wind turbines. Then other students in the United States and Indonesia asked questions of the presenting group and made comments about their mathematical model.

the fact that two countries in the data set (Indonesia and Russia) had missing values for wind power capacity. The students assumed this was because the countries chose not to report that data. On the basis of this assumption, the students felt it fair to give these countries a value of zero for that variable. Other groups assumed that these countries had not yet collected the data for that category and were thus more mathematically lenient, giving Indonesia and Russia the average of the other countries' wind power capacity.

Situation D (see figure 4):

Indonesian Student 4: I was wondering why you gave the lowest proportion to population and more proportions to GDP. Is there any reason behind the proportion of your model?

US Student 5: We weighted GDP more because they have more money to give us as a company. It is kind of a company being, needing to have, profits essentially. And the population has just little less, because the more people there are, the more power they need to support them.

Indonesian Teacher: Ah, OK. So, I think we agree with this amount for wind turbines, but I think your GDP is a bit high [*laughs*]. Sorry!

US Student 6: So, another reason for the GDP being added is because—we talked about this in our discussion—is that if they . . . even though we are giving these windmills for free, we hope that they will come to buy more if they approve of the power they are getting from it and they like the company and that's their thing to buy more. So, that's why we included the GDP. And also for future endeavors with that country and the company.

In situation D, we see the students questioning another group's mathematical model and students sharing their reasons, connected to real-world issues, for those decisions. As teachers of mathematics, we have come to value these kinds of experiences for our students because we find that our students enter a much higher level of engagement and mathematical creativity. Using real data about an important global context and the accountability that comes with justifying ideas to others, our students found an enjoyable challenge that enhanced their connections between mathematical ideas and between mathematics and human endeavors. We desire this high level of engagement for our students each day of their learning.

Fig. 4

Sample Student Models: US Model 1

Find each countries % of the total for the following:
1) CO_2
2) GDP
3) Population
4) WPCapacity

Chile	2
China	125
Indonesia	5
Mexico	16
New Zealand	1
Russia	18
Thailand	5
United States	88

% represents the percent of data from one country between the eight available countries.

Total number of turbines=((0.4*(%CO_2))+(0.3*(%GDP)) +(0.25*(%Population)) +(0.05*(%WPCapacity)))*260

Unlike the group that presented US Model 2, other groups assumed that some countries had not yet collected missing data and gave Indonesia and Russia the average of the other countries' wind power capacity rather than zeros.

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CONNECTING MATHEMATICS TO GLOBAL CITIZENSHIP

This modeling task allowed us to engage our students in thought-provoking mathematical discussions involving important global issues. From the global citizenship perspective, students were able to employ mathematics to discuss critical issues with people from another country who share the same concerns. Students justified the reasoning undergirding their mathematical model and communicated that reasoning to people of another culture. Students from both countries worked through issues of fairness by mathematically attending to which assumptions were acceptable, deciding what variables are important and which ones should be excluded, reasons for giving some variables more weight than others, and which variables were directly or inversely proportional to the related quantity of wind turbine.

The Borean Wind Systems task offered our students an experience of all four of the 21st-century skills—critical thinking and problem solving, collaboration, communication, and creativity (Trilling and Fadel 2009). These are essential skills needed to thrive in the current globalized society. Mathematically, the Borean Wind System modeling task acted as an example of a worthwhile mathematical task, engaging our students in a wide array of mathematical knowledge that is necessary for authentic assessment (Lajoie 1995).

We valued working with teachers and students from another country on issues facing humanity because it allowed critical conversations to occur in our mathematics classrooms. These cross-border conversations deepened our students' engagement in mathematics and encouraged them to strengthen their own justifications. Our students introspectively considered their own beliefs about business and global issues. Wrestling with these mathematical and societal challenges invigorated our students' thinking and learning. We encourage other teachers to connect their classroom and mathematical modeling endeavors with a teacher of a different culture. Using today's technology makes this more possible than ever before, and the rewards of learning mathematics with reasoning and empathy may be the most important fruit we can grow within our students.

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