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Devising a Mathematical Model to Represent College Football Rankings

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Model It!

Devising a Model to Represent College Football Rankings

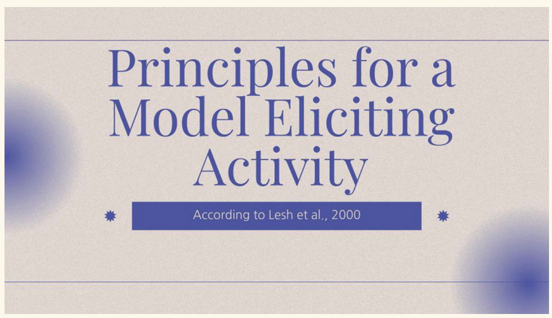
A model eliciting activity based on our students' outside interests sparks engagement with modeling and interesting debates.

Rachel Wiemken, Gabriel Matney, and Brandon Floro

As teachers, we always have an ear to the ground to listen to what our students are discussing, so that we might have insights into their interests. We seek to leverage our students' interests into mathematical tasks for them to deepen their learning and appreciation of mathematics. Every year we notice our students discussing the NCAA's Division 1-A football rankings. We devised a mathematical modeling task in which students could continue their excited conversation about the often-controversial rankings, using mathematics to justify their claims. Our task takes students on a journey to consider the complexity of determining which college football teams should be ranked in the top 10 fairly, with as little bias as possible. They soon discover this is not an easy task, but it is an engaging one. We will describe how the task implementation elicited rich mathematical discourse and produced realistic mathematical models for our students.

This *Model It!* task is a model eliciting activity (MEA) that we developed with the six MEA principles in mind (Lesh et al., 2000). In Video 1, the six principles are explained in detail and how this task is aligned with them. These principles undergirded our process and intentional development of the *Model It!* task, and we hope sharing them illuminates the reader's understanding of the task and expectations set forth for students.

Video 1 Six Principles Related to the MEA Task



Watch the full video online

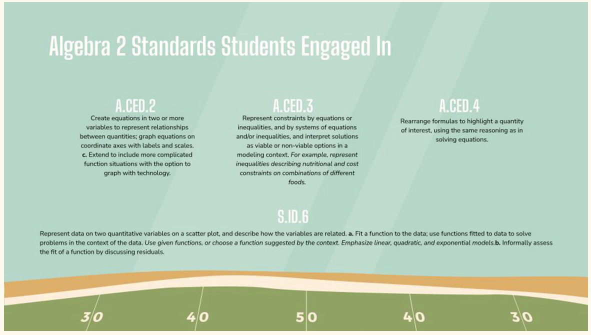
STUDENT PRIOR KNOWLEDGE

This task has been given to seventh-grade students during a genius-hour elective, to 9–12 grade students (in Algebra, Geometry, Algebra 2, Precalculus, and Data Science), and to preservice teachers in a collegiate course. Its design enables students to leverage the prior mathematical knowledge they bring with them and build on it. This work includes mathematics ranging from numbers and operations to algebra to intricate statistical analysis. When we work with younger students who have no modeling experience, we give them some idea about modeling by engaging them in a critique of the viability of a simplistic model. For example, the seventh-grade students look at the data, and we ask them to consider a model that adds up all the values for these statistics to find the largest value and declare the largest value the best football team. Would doing so be a fair way to determine team rankings? Students often note that this would not work for a variety of reasons. From here, students have been primed to think about what a model is and whether it's fair, which launches them into creating their own mathematical model.

STANDARDS ALIGNMENT

This task incorporates several State Standards for Mathematical Practice, but it primarily focuses on modeling with mathematics, which is the fourth standard for mathematical practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Additionally, this task has multiple content standards embedded throughout, in which students can apply numerous algebra and statistical concepts. For more information on the standards alignment, please watch Video 2.

Video 2 Standards Related to the MEA Task



Watch the full video online

IMPLEMENTING THE TASK

We carefully designed the task to prime students' thinking before presenting them with the data. During each phase, students are provided time in their groups to ponder and discuss the task. We recommend group sizes of between two and four students.

Phase 1: How Might We Rank Teams?

In this portion of the task, we provided an entry point for all students. Our goal was to have students brainstorm ways they would rank almost anything. If students struggled to relate to the context of sports, we engaged them in a context they know more about (e.g., bands, video games, cars). Sample criteria responses from students included the following: wins and losses, strength of teams played, strength of individual players, and points given up and points prevented. Ultimately, we wanted students to recognize

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Phase of the Task	MEA Principle(s)	Student Directions
1: How might we rank teams? (10 minutes)	#2 (Reality)	If you were on a committee for a collegiate team sport, what would be the fairest way to rank the top 25 teams?
2: NCAA Football Context (10 minutes)	#2 (Reality)	The current system in place for Division 1-A college football is a four-team bracket, in which two initial games are played and the winners play one another for the national title. The rankings are determined by the college football playoff committee, which is made up of 13 people with a strong football background, who are selected every year to use their expertise to rank the top 25 teams. **What criteria do you think these committee members are using to rank the teams?**
3: Considering Transparency (5 minutes)	#2 (Reality)	Currently, each committee member chooses their own criteria, and it is not known to the public. Do you think this is fair?
4: Model Creation (45–60 minutes)	#1 (Model Construction), #2 (Reality), #3 (Self-Assessment), #4 (Model Documentation), #5 (Model Shareability and Reusability), #6 (Effective Prototype)	**Objective: Determine the top 10 teams mathematically.** The data in the table provided come from reputable sources. For your convenience, here is a google sheet (link online) with the data for 20 teams. Use the data to mathematically rank the best 10 teams of the 2021 season.
5: Presentations (30–45 minutes)	#6 (Effective Prototype)	Students present their mathematical models, assumptions, and explanations to the class.

that they needed some type of criteria to rank the teams.

Phase 2: NCAA College Football Context

In this phase, we narrowed the scope of the context students previously explored. Some were not familiar with the NCAA rankings system, so we used this opportunity to provide the information they needed to be able to complete the task. We prompted students to think about team rankings directly pertaining to the NCAA context. This was an exciting part of the task because there were students who knew a considerable amount about this context. It provided them an opportunity to share their expertise with the class. Sample student responses to this question was included the following: win-loss ratio, strength of schedule, revenue brought in, players recruited, coaching staff, passing and running yards obtained, passing and running yards given up, whether they are in a conference, and so much more!

Phase 3: Considering Transparency

Next, we revealed one more layer to the college football rankings and asked a very simple question about “fairness.” Every time students engaged in this task, all students gave their opinion, even if they did not know football well. All students had life experiences that allowed them to weigh in on the idea of what is fair and what is not. This part of the task was especially enjoyable because even students unfamiliar with the context see a clear problem that they want to fix, which meets the criteria for the second principle for an MEA: the Reality Principle (Lesh et al., 2000). At this point in the task, students were *invested*.

Phase 4: Model Creation

Now, students officially had the full context, data sheet, and directions. This phase is reserved for students to apply the previous whole-class conversations to the specific mathematical model they wanted to build.

This relates to the first principle for an MEA: the Model Construction Principle (Lesh et al., 2000). We gave students ample time, or as much time as we could, to play with the data and mathematical relationships they desired to create.

As students worked through this phase, we monitored and promoted their thinking. At times, students were unsure of where to start, so we prompted them to consider which variables they considered to be most important vs. least important, and why. When students felt confident with the variables they chose but struggled to decipher how the numbers would interact with one another, we prompted them to think about the different mathematical models they had engaged with in the past. We reminded students this could be from this school year or prior. Once students had a grasp of examples of mathematical models they had worked with, they felt confident in pursuing a path knowing they could refine their model at any point. During moments when students claimed to be done with much time left to work, we asked numerous questions to help students assess the reasonableness of their model and/or asked students to apply their model to a different season and see if the results hold true.

Overall, this phase addresses four principles of a MEA (Lesh et al., 2000). It again demonstrates the third MEA principle (Self Assessment Principle) because students are constantly assessing the reasonableness of their model. They wanted it to be as *fair* a model as it could be and strived to be more fair than the current model. This phase also meets the criteria of the fourth principle of a MEA (Model Documentation Principle) because students must create a presentation explaining assumptions they made and explain the creation and interactions of their mathematical model. This phase also meets the criteria of the fifth MEA principle (Model Share-ability and Reusability), as students have the opportunity to extend their thinking to applying their model to another data set. Lastly, this phase engages the sixth MEA principle (Effective Prototype Principle) because students need a model that is simple enough to explain to their classmates yet accurately ranks the teams. The students mentioned to us that they could apply their models to other contexts, such as other sports, video games, and other related data they knew about.

Phase 5: Presentations

Then, students shared the mathematical models they created and the resulting rankings. These presentations are designed to be a pitch to the college football playoff committee proving that the model they created is fairer than the current model in use. At the end of presentations, the teacher and other students could give feedback on the presenter's model. From prior experiences, students make comments on aspects of the model they particularly like and ask probing questions, typically, to test the validity of the model. The teacher also facilitates conversations and asks their own questions, some of which are included on the main task page. This last phase contributes to the MEA principles because students are explaining how they worked through each principle during their model creation time.

SAMPLE STUDENT MODELS AND PRESENTATIONS

In the supplementary material, we have included a diverse set of student mathematical models and presentations, ranging from Grade 7 through the collegiate level. Model 1 (link online) represents an algebraic perspective on the task, while Model 2 (link online) focuses on a more statistical methodology. In Videos 3 and 4, students also take an algebraic approach; however, the videos include their explanation and their response to questions.

Video 3 Student Presentations

Watch the full video online

Video 4 Student Presentations

Watch the full video online

CONCLUSION

This task was born out of our students' interests and invokes rich, realistic, mathematical work and conversations. We implemented this task numerous times with middle school, high school, and collegiate students, all yielding different mathematical models that resolve the problem posed in the task. We share this task with readers so they can have similar experiences with their students. In addition, for those who like this type of model eliciting activity, check out our Borean Winds MEA (Wiemken et al., 2021). We hope your students enjoy this task as much as ours have, and we would love to see the mathematical models your students create!

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Model It! Representing Football Rankings

Situation:

Other coaches and the general public have been pushing for years for the CFP (College Football Playoff) Committee to be transparent about the way they determine their rankings. This year you have been selected as one of the lucky 13 members for the CFP Committee. It is requested that you share your method for ranking the **top 10** teams. You know that if you create a mathematical model to rank the teams, there is less room for argument, because your decision is based on data. Other members on the committee agree that this is the best method and want to work with you to create this mathematical model.

Objective: Rank the best 10 teams of the 2021 season mathematically, using the data provided in the table. The data comes from reputable sources which are linked in the table. For your convenience, we have generated a google sheet with the data for 20 teams from the 2021 season. DATA SHEET (link online).

Presentation: You will create a presentation to share with the class justifying why your mathematical model and rankings are the fairest. Below are the items to include in your presentation.

1. Display your table of teams showing the number 1 team on top all the way to the number 10 team.
2. Create a mathematical model to represent your rankings.
3. Explanation of how you used data to determine the rankings.
4. List any assumptions you needed to make in the process.

Looking for More? Below are questions to consider as you develop your model and presentation. These questions can be addressed in your presentation if you choose and, additionally could be discussion questions addressed after your presentation.

1. Could anyone argue that other teams should be in the top four instead of the teams your model selects? Why or why not?
2. What separates the number 10 team from the number 11 team? Explain your reasoning.
3. If your favorite team was in the top 20 teams of the original data, explain the difficulty of not including your bias to input your favorite team in the top 10.
4. What discussions were had among your group of which data from the table to include and which to not include? Were there any disagreements about the data to use, and how did you resolve those disagreements?